Maciej Ławryńczuk

Nonlinear Predictive Control Using Wiener Models

Computationally Efficient Approaches for Polynomial and Neural Structures

September 29, 2021

Springer Nature

Please cite the book:
Maciej Ławryńczuk: Nonlinear Predictive Control Using Wiener Models: Computationally Efficient Approaches for Polynomial and Neural Structures. Studies in Systems, Decision and Control, vol. 389, Springer, Cham, 2022.

E-mail: M.Lawrynczuk@ia.pw.edu.pl

To my children, young scientists

Foreword

And another new book . . .

Aren't there already so many books about Model Predictive Control (MPC)?

Is not already everything explained in Wikipedia?

Maybe. The monograph written by professor Maciej Ławryńczuk goes a different way than usual, does not repeat everything that has been written many times anywhere. Aiming at application to a large class of nonlinear SISO and MIMO systems approximable by well-known Wiener models, professor Ławryńczuk presents his approaches and procedures developed over the years, which are ideally suited for modern, complex, nonlinear tasks in control and automation engineering.

For several decades, model predictive control has been one of the promising mathematically-based but strongly algorithmic research branches of modern control engineering. After and besides the phases of research with rigorous mathematics research with the goal to solve control tasks of nonlinear systems and processes by nonlinear control with final proof of stability, convergence, and error behavior, also mathematical-based approaches for the same problem classes but with more complex requirements e.g. concerning manipulated variable constraints, local model approximations and optimization characterized by (programmable) algorithms have been developed in the last decades. Besides the developments of the so-called model-free control or model adaptive control, model predictive control is characterized by a very high adaptability to temporal and physical local conditions of the system to be controlled.

The development in the last years as well as the specificity of the approach regarding the prediction of the controlled system behavior with simultaneous optimization or search for the suitable local control strategy led to the development of more and more complex MPCs with more and more complex algorithms. Accordingly, typical fields of application are systems or processes with a related slow dynamics. The complexity of the plant dynamics, the complexity of the MPC, the plant dynamics as well as the available possibilities of the microprocessor or computer hardware finally determine the technical feasibility. Consequently, in the last decades, the development of computationally expensive algorithms dominates.

With this background as motivation and facing the goal to develop numerically efficient algorithms for nonlinear SISO and MIMO plants, professor Maciej Ławryńczuk's monograph addresses an alternative path. His detailed view for numerical efficiency down to the equation level using online (nowadays also denoted as data-driven) modeling and also trajectory linearization combined with parameterization of decision variables using the Laguerre functions reduces the complexity of the MPC optimization task.

A few structures of Wiener models are discussed for input-output and state space systems. The author prefers to use Wiener models with neural static part and is able to show why. The author's MPC algorithms based on underlying neural Wiener models show better performance and robustness with respect to modeling errors and disturbances than classical MPC approaches based on inverse static models. Based on this in detail explained core, efficient algorithms as highly effective alternatives are developed which finally leads to a textbook for both fundamental researchers and implementation-oriented practitioners.

Besides parametrizing the approximating models for short, computationally efficient horizons, the author uses two complex application examples, presented and developed in detail, to impressively demonstrate the presented MPC approaches and setting parameters and resulting performance.

Yes, there are many books on MPC and its diverse manifestations but only a few approaches that pursue the author's addressed goals of computationally efficient realization with the goal of applications to practical unknown nonlinear systems. The author shares his knowledge fully and in detail, enabling the reader to follow (and thus develop) the approaches in detail and apply them directly. Very impressive. Highly recommended reading. Perhaps someone has to add this book with the described potential to allow many new MPC realizations . . . also to Wikipedia as a new standard.

Diusburg, May 2021 *Dirk Söffker*

Preface

Good control is necessary for economically efficient and safe operation of various processes, including industrial plants, e.g. distillation columns, chemical reactors or paper machines, and processes with embedded control systems, e.g. drones or autonomous vehicles. The Model Predictive Control (MPC) methodology is a very powerful tool which may be used to control complicated processes. MPC is an advanced control method in which a dynamical model of the process is repeatedly used on-line to predict its future behaviour and an optimisation procedure finds the best control policy. Typically, MPC algorithms lead to much better control quality than the classical Proportional-Integral-Derivative (PID) controller, particularly in the case of Multiple-Input Multiple-Output (MIMO) processes with strong cross-couplings, also with delays, and when some constraints must be imposed on process variables. Numerous classical MPC algorithms for processes described by various linear models have been developed over the years; they are widely used in practice.

Many processes are inherently nonlinear. In such cases, the rudimentary MPC algorithms which use linear models are likely to result in unacceptable control quality or even do not work. This book aims to present a few computationally efficient nonlinear MPC algorithms for processes described by input-output and state-space Wiener models defined by a serial connection of a linear dynamic block followed by a nonlinear static one. The considered class of models can approximate properties of many processes very well using a limited number of parameters. Furthermore, due to the Wiener models' specialised structure, implementation of the presented MPC algorithms is relatively uncomplicated. For two technological processes, i.e. a neutralisation reactor and a proton exchange membrane fuel cell, effectiveness of polynomial and neural Wiener models is thoroughly compared.

The key issue in this book is computational efficiency of MPC. When a nonlinear model, including the Wiener model, is used for prediction in MPC, a nonlinear constrained optimisation problem must be solved at each sampling instant on-line. In order to reduce computational complexity and computation time, two concepts are used. Firstly, a few approaches using on-line model or trajectory linearisation are possible. As a result, relatively simple quadratic optimisation tasks are obtained (they have only one global solution), nonlinear on-line optimisation is unnecessary. Secondly, parameterisation of the computed decision variables using Laguerre functions is possible to reduce the number of actually optimised variables.

This book consists of nine chapters. It also includes the list of symbols and acronyms used, the lists of references and the index.

Chapter 1 is an introduction to the field of MPC. Its basic idea and the rudimentary MPC optimisation problems are defined, parameterisation of the decision variables using Laguerre functions is described. A literature review on computational complexity issues of nonlinear MPC is given, many example applications of MPC algorithms in different fields are reported.

Chapter 2 describes the considered structures of Wiener models: six input-output configurations and three state-space ones. A short review of identification methods of Wiener models is given, possible internal structures of both model parts are discussed and example applications of Wiener models are reported. Alternative structures of cascade models are mentioned.

Chapter 3 details MPC algorithms for processes described by input-output Wiener models: the classical simple MPC method based on the inverse static model, the rudimentary MPC algorithm with nonlinear optimisation, two MPC schemes with on-line model linearisation and two MPC methods with advanced trajectory linearisation. Variants of all algorithms with parameterisation using Laguerre functions are also described.

Chapter 4 thoroughly discusses implementation details and simulation results of the considered MPC algorithms applied to five input-output benchmark processes. Different features of the algorithms are emphasised, including set-point tracking ability and robustness when the process is affected by disturbances and modelling errors. All algorithms are compared in terms of control quality and computational time.

Chapters 5 and 6 compare effectiveness of different Wiener models' configurations to approximate properties of two simulated technological processes: a neutralisation reactor and a proton exchange membrane fuel cell. Polynomials and neural networks are used in the nonlinear static block of the models. Properties of both model classes are thoroughly discussed. Next, simulations of various MPC algorithms are presented. A few variants of constraints imposed on the predicted value of the controlled variable, including soft approaches, are considered for the neutralisation reactor.

Chapter 7 details variants of all MPC algorithms presented in Chapter 3 for processes described by state-space Wiener models. The classical and an original, very efficient prediction method, which allow for offset-free control, are presented.

Chapter 8 thoroughly discusses implementation details and simulation results of the considered MPC algorithms applied to three state-space benchmark processes. In particular, efficiency of different methods allowing for offset-free control is compared.

Chapter 9 summarises the whole book; some future research ideas are also given.

This book is intended to be useful for everyone interested in advanced control, particularly graduate and PhD students, researchers and practitioners who want to implement nonlinear MPC solutions in practice.

Warsaw, May 2021 *Maciej Ławryńczuk*

Acknowledgements

I acknowledge the permission of Elsevier to reproduce portions of the following journal article: Ławryńczuk, M., Tatjewski, P.: Offset-free state-space nonlinear predictive control for Wiener systems. *Information Sciences*, vol. 511, pp. 127–151, 2020.

Warsaw, May 2021 *Maciej Ławryńczuk*

Contents

Notation

General notation

$a, b, \ldots$	variables or constants, scalars or vectors
$\boldsymbol{A}, \boldsymbol{B}, \ldots$	matrices
a^{T}, $\boldsymbol{A}^{\mathrm{T}}$	transpose of the vector a and of the matrix $\boldsymbol{A}$
$\mathrm{diag}(a_1, \ldots, a_n)$	the diagonal matrix with $a_1, \ldots, a_n$ on the diagonal
$\left.\frac{\mathrm{d}y(x)}{\mathrm{d}x}\right\vert_{x=\bar{x}}$	the derivative of the function $y(x)$ at the point $\bar{x}$
$\left.\frac{\partial y(x)}{\partial x_i}\right\vert_{x=\bar{x}}$	the fractional derivative of the function $y(x) = y(x_1, \ldots, x_{n_{\mathrm{x}}})$ with respect to the scalar x_i at the point $\bar{x}$
$f(\cdot)$, $g(\cdot), \ldots$	scalar or vector functions
$\mathbf{0}_{m\times n}$, $\boldsymbol{I}_{m\times n}$	zeros and identity matrices of dimensionality $m \times n$
q^{-1}	the discrete unit time-delay operator
$\Vert x\Vert_{\boldsymbol{A}}^2$	$x^{\mathrm{T}}\boldsymbol{A}x$

Processes and models

a_i, b_i, a_i^m, $b_i^{m,n}$	the parameters of the linear dynamic part of the input-output Wiener model
$a_{i,j}$, $b_{i,j}$, $c_{i,j}$	the parameters of the linear dynamic part of the state-space Wiener model
$\boldsymbol{A}(q^{-1})$, $\boldsymbol{B}(q^{-1})$	the polynomial matrices that describe the linear dynamic part of the input-output Wiener model
$\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$	the matrices that describe the linear dynamic part of the state-space Wiener model
$g(\cdot)$, $g_m(\cdot)$	the functions that describe the nonlinear static blocks of the Wiener model
$\tilde{g}(\cdot)$, $\tilde{g}_m(\cdot)$	the functions that describe the inverse models of the nonlinear static blocks of the Wiener model

k	the discrete time sampling instant ($k = 0, 1, 2, \ldots$)
K	the number of hidden nodes in a neural network, the degree of a polynomial
$n_{\mathrm{A}}, n_{\mathrm{B}}, n_{\mathrm{A}}^{m}, n_{\mathrm{B}}^{m,n}$	the constants that define the order of dynamics of the linear dynamic part of the input-output Wiener model
n_{u}	the number of inputs (manipulated variables)
n_{v}	the number of outputs of the linear dynamic block of the Wiener model
n_{x}	the number of state variables
n_{y}	the number of outputs (controlled variables)
$u(k)$	the input vector at the sampling instant k
$v(k)$	the vector of outputs of the linear dynamic block of the Wiener model at the sampling instant k
$x(k)$	the state vector at the sampling instant k
$\tilde{x}(k)$	the estimated state vector at the sampling instant k
$y(k)$	the output vector at the sampling instant k
$\triangle u(k+p\|k)$	$u(k+p\|k) - u(k+p-1\|k)$

MPC algorithms

$d(k)$	the vector of unmeasured output disturbances at the sampling instant k
$\boldsymbol{G}(k)$	the step-response matrix of the model linearised at the sampling instant k
$\overline{\boldsymbol{G}}$	the constant step-response matrix of the linear dynamic part of the Wiener model
$\boldsymbol{H}(k), \boldsymbol{H}^{t}(k)$	the matrix of derivatives of the predicted output trajectory with respect to the future input trajectory at the sampling instant k
$J(k)$	the cost-function minimised in MPC
$K(k), K_m(k), K_{m,n}(k)$	the gains of the nonlinear static part of the Wiener model at the sampling instant k
N	the prediction horizon
N_{u}	the control horizon
$s_p(k), s_p^{m,n}(k), \boldsymbol{S}_p(k)$	the step-response coefficients and the step-response matrix of the model linearised at the sampling instant k for the sampling instant p
$\bar{s}_p, \bar{s}_p^{m,n}, \overline{\boldsymbol{S}}_p$	the constant step-response coefficients and the constant step-response matrix of the linear dynamic part of the Wiener model for the sampling instant p
$u(k+p\|k)$	the process input vector calculated for the sampling instant $k+p$ at the sampling instant k
$\boldsymbol{u}(k)$	the process input vector calculated at the sampling instant k over the control horizon

$u^{\min}$, $u^{\max}$	the vectors of magnitude constraints imposed on process inputs
$\boldsymbol{u}^{\min}$, $\boldsymbol{u}^{\max}$	the vectors of magnitude constraints imposed on process inputs over the control horizon
$v(k+p\|k)$	the output vector of the linear dynamic part of the Wiener model predicted for the sampling instant $k+p$ at the sampling instant k
$\boldsymbol{v}(k)$	the output vector of the linear dynamic part of the Wiener model predicted at the sampling instant k over the prediction horizon
$x(k)$, $\tilde{x}(k)$	the vectors of measured and estimated state variables at the sampling instant k
$x^0(k+p\|k)$	the state free trajectory vector predicted for the sampling instant $k+p$ at the sampling instant k
$\boldsymbol{x}^0(k)$	the state free trajectory vector predicted at the sampling instant k over the prediction horizon
$\hat{x}(k+p\|k)$	the state trajectory vector predicted for the sampling instant $k+p$ at the sampling instant k
$\hat{\boldsymbol{x}}(k)$	the state trajectory vector predicted at the sampling instant k over the prediction horizon
$y^0(k+p\|k)$	the output free trajectory vector predicted for the sampling instant $k+p$ at the sampling instant k
$\boldsymbol{y}^0(k)$	the output free trajectory vector predicted at the sampling instant k over the prediction horizon
$\hat{y}(k+p\|k)$	the output trajectory vector predicted for the sampling instant $k+p$ at the sampling instant k
$\hat{\boldsymbol{y}}(k)$	the output trajectory vector predicted at the sampling instant k over the prediction horizon
$y^{\min}$, $y^{\max}$	the vectors of magnitude constraints imposed on predicted output variables
$\boldsymbol{y}^{\min}$, $\boldsymbol{y}^{\max}$	the vectors of magnitude constraints imposed on predicted output variables over the prediction horizon
$y^{\mathrm{sp}}(k+p\|k)$	the output set-point trajectory vector for the sampling instant $k+p$ known at the sampling instant k
$\boldsymbol{y}^{\mathrm{sp}}(k)$	the output set-point trajectory vector at the sampling instant k over the prediction horizon
$\triangle u(k+p\|k)$	the vector of input increments calculated for the sampling instant $k+p$ at the sampling instant k
$\triangle \boldsymbol{u}(k)$	the vector of input increments calculated for the sampling instant k over the control horizon (the vector of decision variables calculated in MPC)
$\triangle u^{\min}$, $\triangle u^{\max}$	the vectors of constraints imposed on increments of the input variables
$\triangle \boldsymbol{u}^{\max}$	the vectors of constraints imposed on increments of the input variables over the control horizon

$\varepsilon^{\min}(k)$, $\varepsilon^{\max}(k)$	the vectors that define the degree of hard output constraints' violation constant over the prediction horizon
$\varepsilon^{\min}(k+p)$, $\varepsilon^{\max}(k+p)$	the vectors that define the degree of hard output constraints' violation varying over the prediction horizon
$\underline{\boldsymbol{\varepsilon}}^{\min}(k)$, $\underline{\boldsymbol{\varepsilon}}^{\min}(k)$	the vectors that define the degree of hard output constraints' violation over the prediction horizon
λ, $\lambda_{n,p}$, $\boldsymbol{\Lambda}$, $\boldsymbol{\Lambda}_p$	the weighting coefficients and the weighting matrices related to control increments
$\mu_{m,p}$, $\boldsymbol{M}$, $\boldsymbol{M}_p$	the weighting coefficient and the weighting matrices related to the predicted output control errors
$\nu(k)$	the vector of unmeasured state disturbances at the sampling instant k
$\rho^{\min}$, $\rho^{\max}$	the weighting coefficients of the penalties related to violation of hard output constraints

Acronyms

DMC	Dynamic Matrix Control
GPC	Generalized Predictive Control
IMC	Internal Model Control
LMPC	MPC algorithm based on a linear model
LS-SVM	Least Squares Support Vector Machine
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MLP	Multi-Layer Perceptron feedforward neural network
MPC	Model Predictive Control
MPC-NO	MPC algorithm with Nonlinear Optimisation
MPC-NO-P	MPC-NO algorithm with Parameterisation
MPC-NPLPT	MPC algorithm with Nonlinear Prediction and Linearisation along the Predicted Trajectory
MPC-NPLPT-P	MPC-NPLPT algorithm with Parameterisation
MPC-NPLT	MPC algorithm with Nonlinear Prediction and Linearisation along the Trajectory
MPC-NPLT1	MPC-NPLT algorithm with linearisation along the trajectory defined by the input signals applied at the previous sampling instant
MPC-NPLT2	MPC-NPLT algorithm with linearisation along the trajectory defined by the optimal input signals calculated at the previous sampling instant
MPC-NPLT-P	MPC-NPLT algorithm with Parameterisation
MPC-NPSL	MPC algorithm with Nonlinear Prediction and Simplified model Linearisation for the current operating point
MPC-NPSL-P	MPC-NPSL algorithm with Parameterisation

MPC-SSL	MPC algorithm with Simplified Successive model Linearisation for the current operating point
MPC-SSL-P	MPC-SSL algorithm with Parameterisation
MPC-inv	MPC algorithm based on the inverse model of the nonlinear static part of the Wiener system
PID	Proportional-Integral-Derivative controller
RBF	Radial Basis Function feedforward neural network
SISO	Single-Input Single-Output
SQP	Sequential Quadratic Programming
SVM	Support Vector Machine