



Practice article

Control quality assessment using fractal persistence measures

Paweł D. Domański

Warsaw University of Technology, Institute of Control and Computation Engineering, ul. Nowowiejska 15/19, 00-665 Warszawa, Poland

HIGHLIGHTS

- Control error histogram is often non-Gaussian with fat-tail properties.
- Fractal approach is robust against disturbances.
- Persistence measures are useful in control quality assessment.
- Multi-persistence Hurst exponents and crossover increase CPA applicability.

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ABSTRACT

Control Performance Assessment (CPA) has great practical importance. Control quality significantly affects final production throughput, efficiency and environmental impact. There are many approaches starting from time-domain methods, through Minimum Variance, Gaussian and non-Gaussian statistics up to alternative wavelet, fractal or entropy measures. Analysis of production data from process industry shows that signals are often described by non-Gaussian distributions, mostly fat-tail. On the other hand, simulations show that strong disturbances may significantly screen ability of proper detection. This work tests different approaches, i.e. Gaussian standard deviation and fat-tail distribution factors, integral indexes and focuses on persistence measures of rescaled range R/S plot. Robustness of above measures against disturbances with varying statistical properties is investigated. Results confirm that fractal measures may be applied as robust alternative to standard statistics.

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1. Introduction

Control quality plays crucial role in production performance. Especially now, the issues of product throughput maximization, production safety, environmental protection or energy management push systems towards their constraints. They force operation close to the edge of technological limitations. Moreover, real installations are mostly non-stationary, time varying and complex systems with many internal, unknown, coupled correlations. Long delays and human factor complement the overall picture [1]. Single shot implementation and tuning mostly solves problems only in short-term perspective. Actually, high quality production requires constant attention and close supervision to sustain high performance numbers [2,3].

Complexity of industrial processes challenges the CPA task. The process nature not only affects control system setpoint tracking and filtering abilities, but also hampers algorithms and measures used to detect control system quality. This effect is visible in the review of industrial control loops. It has been shown [4] that large majority of process PID loops does not witness Gaussian properties

having evident fat-tails. It further causes that loop quality measures assuming signal normal distributions may not be appropriate. The need for new more suitable indexes appears.

There are two main reasons. Majority of industrial control loops is poorly tuned or uses improper philosophy [5]. It clearly affects control signals Probabilistic Distribution Function (PDF) broadness. Second reason lies in the nonlinearities, disturbances and complexity of the process itself. Cross-correlated feedbacks and noises cannot be fully decoupled, so their impact exists. Further consequences are associated with varying long-range correlations of the small and large fluctuations. They cause persistence effects, i.e. Hurst exponent deviating from 0.5 [6]. Complex process dynamics frequently brings about multiple scaling exponents in the same range of scales [7]. Thus, one may additionally expect multi-persistence behavior.

The main challenge of the approach is to propose the alternative methodology that may be effectively used in case of the non-Gaussian loop properties. Majority of the existing approaches assumes Gaussian properties in the loop. Once the loop witnesses other disturbances the method does not have to be optimal or even may lead to the unclear assessment. Actually, there is limited number of publications addressing non-Gaussian loops [8–10]. Rescaled range R/S analysis offers tools addressing similar issues.

E-mail address: p.domanski@ia.pw.edu.pl.

Fractal approach has already met acceptance in many areas of research. Starting from economics it has been successfully applied in physics, biology, medicine, computer graphics, network traffic analysis, telecommunication, time series prediction, etc. It offers several tools, mostly based on persistence and crossover properties [11]. It appears that the transfer of these methods to the control engineering allows for further insight into the loop properties.

It has been shown that classical methods may be strongly biased by the non-Gaussian disturbances and properties. Fractal and persistence measures enable to cope with that problem and give another insight into the loop properties. It enables not only to detect poor performance but also shows what is the loop quality disadvantage (aggressive versus sluggish control) giving hints what should be done to improve loop quality. Further, the existence of the fat tails and persistence properties inform about the uncoupled disturbances thus suggesting the need for loop structure modification with decoupling and feedforward.

The first research on the use of the Hurst exponent [12–14] for control quality assessment has stimulated the further work towards the analysis of the multi-persistence, multiple Hurst exponents, crossovers, etc. This area in control engineering context is still not exploited much, in contrary to the other research areas, like physics, medicine, economy or social sciences and this work tries to fill this gap.

The usage of the fractal methods in control engineering requires multidisciplinary research and review of the achievements done in other research contexts, like mathematics, statistics, physics and even economics. As fractal approach has been successfully used in other research areas, the results has to get new, CPA, interpretation. Additionally the presented CPA methodology brings into picture new interpretation of the outliers. In all classical approaches the outliers are considered as something wrong and undesirable. The information brought by them is ignored. The use of fractal methods considers them seriously and tries to use it.

The paper shows adoption of persistence analysis to the engineering task of control quality assessment. The problem is formulated and basic measures are introduced. Analysis is focused on the PID loop example. Evaluation starts with histogram analysis, followed by R/S plot review and the persistence measures. Paper closes with the comparison of performance detection quality impeded by disturbances. Final section concludes paper with results discussion and presentation of issues open for further research.

2. Control loop performance assessment

Control systems often do not perform efficiently. It is caused by several internal and external reasons. We may enumerate the main ones:

1. insufficient daily control system maintenance caused by process fluctuations, poor tuning, modifications of plant operating conditions, service activities and shortage of experienced personnel,
2. instrumentation problems and failures,
3. unsuitable control philosophy.

Human maintenance is inefficient and does not cover all needs. Researchers and engineers continuously work on development and implementation of automated and autonomous methodologies to evaluate clear quality measures, to prepare straightforward validation procedures, to translate technical numbers into business language and financial benefits, to evaluate methodologies, to estimate and predict possible financial savings due to the process control system upgrade and to design commercially available software solutions.

First CPA project was performed for paper machine in 1967 [15]. Development of tools continued in 70's [16], 80's [17] accelerating

in 1989 with introduction of Minimum Variance (MinVar) [18] approach. The research followed that path covering various aspects of the control system, like univariate feedback [19] and feedforward control [20], unstable and nonminimumphase systems [21], multivariate MIMO cases [22], varying setpoint [23], cascaded control [24], aspects of setpoint tracking versus disturbance rejection [25], linear time-variant systems [26]. There have been evaluated different domain groups of methods in such a long history of research:

1. time-domain indexes based on step test response, i.e. delay, undershoot, rising, maximum, settling time, overshoot and peak amplitude [27],
2. time-domain approaches using normal operation data: Mean Square Error (MSE), Integral of Square Error (ISE), Integral of Absolute Error (IAE), Amplitude Index (AMP) [28,29],
3. minimum variance and model-based indexes [30–32],
4. statistical factors utilizing different probabilistic distribution function (standard deviation, variance, skewness, kurtosis, scale or shape, ...) or various probabilistic distributions [33–36],
5. Gaussian mixture models [37] and Bayesian inference-based finite Gaussian mixture models [38],
6. frequency domain methods [39], like with the use of the Fourier transform [40], sensitivity function [41] or the reference to disturbance ratio (RDR) index [42],
7. novel indexes using wavelets [43], orthonormal Laguerre networks [44] and other functions [5], singular spectrum analysis [45], neural networks [46], support vector machines [47], Hurst exponent [14], persistence measures [48], multifractals [49], entropy [8] and others [9,10],
8. business Key Performance Indicators expressed in monetary units [50].

The presented work focuses on persistence approach and extended range R/S analysis. Indexes are confronted with statistical factors of different PDFs and classical integral indexes. The results are benchmarked with undisturbed ideal measures evaluated from step response, i.e. overshoot κ and settling time T_{set} . Control error signal is used as the variable carrying on information about loop performance. It is independent on setpoint variations with expected mean value equal to zero and there is no need of detrending.

2.1. Performance measures

There are two main integral indexes: MSE and IAE. They are often used alternately. However, it has been shown that tuning with goal to minimize MSE not only punishes large deviations from setpoint, but also generates aggressive control actions [51]. It is also suggested that IAE is closest to economic aspects [52]. Both indexes reflect similar information. IAE seems to be more realistic [53] and thus MSE is excluded from further analysis.

Four statistical factors are used:

- normal (Gauss) standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N - 1}} \quad (1)$$

It informs about signal variability. Higher σ means larger variable variations, while small values means limited fluctuations.

- Lévy α -stable distribution belongs to the family of stable distributions. It has more degrees of freedom (2) and is parametrized by four parameters.

$$PDF_{\alpha,\beta,\delta,\gamma}(t) = \exp\{i\delta t - |\gamma t|^\alpha (1 - i\beta l(t))\}, \quad (2)$$

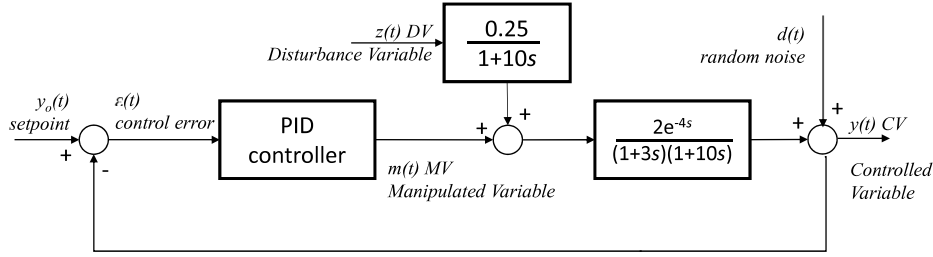


Fig. 1. Close loop simulation environment.

where

$$l(t) = \begin{cases} \operatorname{sgn}(t) \operatorname{tg}\left(\frac{\pi\alpha}{2}\right) & \text{for } \alpha \neq 1 \\ -\operatorname{sgn}(t) \frac{2}{\pi} \ln|t| & \text{for } \alpha = 1 \end{cases},$$

and $0 < \alpha \leq 2$ called stability index, $|\beta| \leq 1$ skewness parameter, $\delta \in \mathbb{R}$ location and $\gamma > 0$ scale factor. In our deliberations two of them are considered: stability parameter α responsible for fat tails and scaling γ reflecting function broadness.

- Laplace distribution, called double exponential, has PDF given as:

$$\text{PDF}_{\mu,b} = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}}, \quad (3)$$

where $\mu \in \mathbb{R}$ is location and $b > 0$ scaling.

Parameter b will be considered as potential measure.

Research shows that fractal behavior may be observed in many areas. Analysis of control engineering time series originating from real industrial systems reveals similar properties. There are several factors allowing to verify fractal hypothesis. One of them is to calculate Hurst exponent H , which measures time series persistence. It is defined as the asymptotic property of the rescaled range R/S function:

$$\left(\frac{R}{S}\right)_n = cn^H, \quad n \rightarrow \infty \quad (4)$$

where S - standard deviations in time slot n , c - positive constant, n - number of observations, H - Hurst exponent. H is calculated using logarithms

$$\ln E(R/S)_n = \ln c + H \ln n \quad (5)$$

plotted in double logarithmic scale $E(R/S)_n$ from n estimating H as linear slope. It may receive following values:

- $H = 0.5$ meaning that all observations are statistically independent and process is stochastically uncorrelated,
- $0 < H < 0.5$ determining anti-persistent time series,
- $0.5 < H \leq 1$ reflecting persistent process,
- for $H > 1$ processes are said to have no dependency in time domain.

The following aspects of R/S plot are used in the analysis: single Hurst exponent $H^{(0)}$ and, with assumption of two scales, $H^{(1)}$ short- and $H^{(2)}$ long-memory exponents separated with single crossover point denoted by n^{cross} .

Concluding, nine factors are analyzed: σ , b , α , γ , IAE, $H^{(0)}$, $H^{(1)}$, $H^{(2)}$ and n^{cross} .

3. Simulations

There have been run specific simulation experiments to validate hypothetical impact of disturbance properties on control quality assessment. Experiments are conducted in the environment

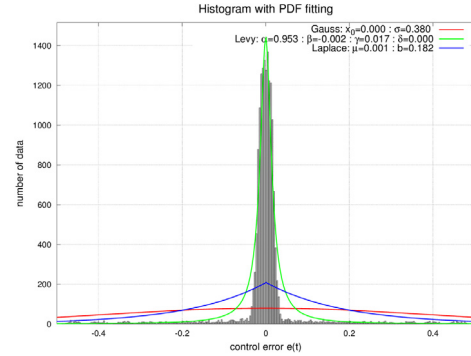


Fig. 2. No disturbance.

presented in Fig. 1. Controlled process is linear and described by second order transfer function with delay and parameters: $K = 2$, $T_0 = 4$, $T_1 = 3$, $T_2 = 10$.

There are two types of disturbances representing industrial situations: random noise $d(t)$ with small magnitude 0.05 and main disturbance added before the plant $z(t)$. Different scenarios for the signal $z(t)$ are considered:

1. no disturbance added, i.e. $z(t) = 0$,
2. Gaussian process with $\mu = 0$, $\sigma = 1.5$ and magnitude $K = 0.75$,
3. α -stable PDF: $\alpha = 1.35$, $\beta = -0.05$, $\gamma = 0.35$, $\delta = 0$ and $K = 0.75$,
4. Laplace PDF with $\mu = 0$, $b = 1.5$ and magnitude $K = 0.75$.

In all cases setpoint is simulated as rectangular wave of period equal to 100 [s] with 20000 samples in total. PID algorithm is used as controller. At first, optimal tuning is determined: $k_p = 0.809$, $T_i = 13.767$, $T_d = 2.86$. It is characterized by overshoot $\kappa = 1.52\%$ and settling time $T_{\text{set}} = 23.6$ [s].

3.1. Histogram analysis

Control error histograms for all disturbance types have been derived. Fig. 2 shows histogram of the non-disturbed loop, Fig. 3 addresses the loop disturbed with the normal PDF disturbance, Fig. 4 with α -stable and Fig. 5 with Laplace PDF. The drawings present fitted distribution functions of different properties.

We notice that none of the histograms is Gaussian. It is due to the long tails associated with two separate operating regimes of the control loop: transient period after setpoint changes and steady state operation covering majority of time. Such character of control errors often happens in real life. Scaling parameter b of Laplace distribution is not ideal. It offers better fitting than Gaussian function, but still there is a place for further improvement. α -stable distribution offers possibility to reflect control error histogram. Stability α and scaling γ factors are considered as effective statistical indexes.

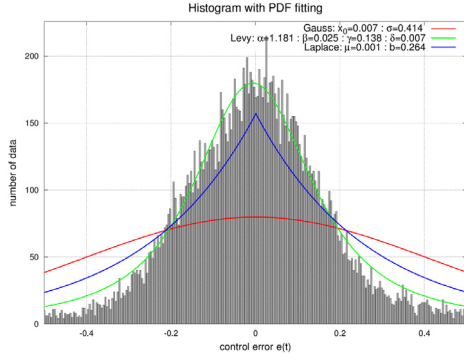


Fig. 3. Normal distribution.

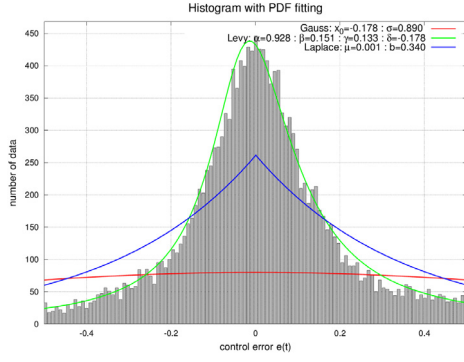
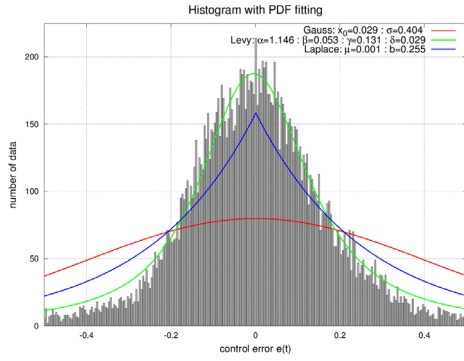
Fig. 4. α -stable distribution.

Fig. 5. Laplace distribution.

Table 1
PID controllers tuning.

Tuning	k_p	T_i	T_d	K	T_{set}
optimal	0.809	13.767	2.86	1.52	23.6
aggressive	1.2	11	2.86	34.1	31.5
sluggish	0.7	18	2.86	0	> 100

3.2. R/S plots

Next stage is associated with review of persistence properties. Different controller settings are applied to four disturbance scenarios (defined in Section 3): non, Gauss, α -stable and Laplace. PID tuning reflects optimal, aggressive and sluggish control (Table 1).

R/S plots are sketched in three graphs. Fig. 6 represents four scenarios presenting different disturbances. Figs. 7 and 8 show sluggish and aggressive tuning.

Persistence parameters, i.e. values of the single Hurst and double memory scale exponent together with the crossover point are

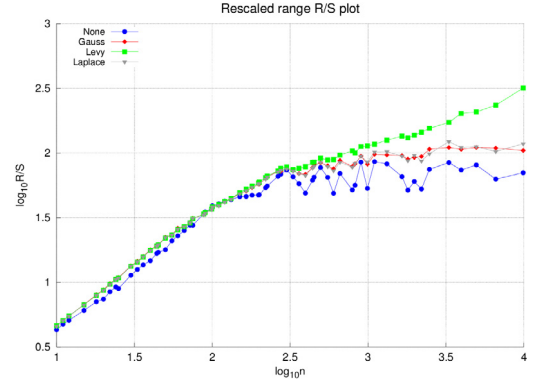


Fig. 6. R/S plot for optimally tuned PID controller.

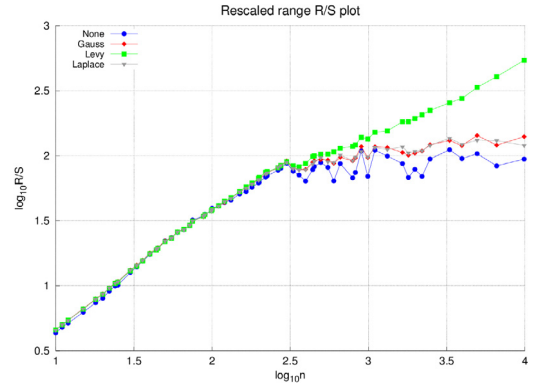


Fig. 7. R/S plot for sluggish tuning of PID controller.

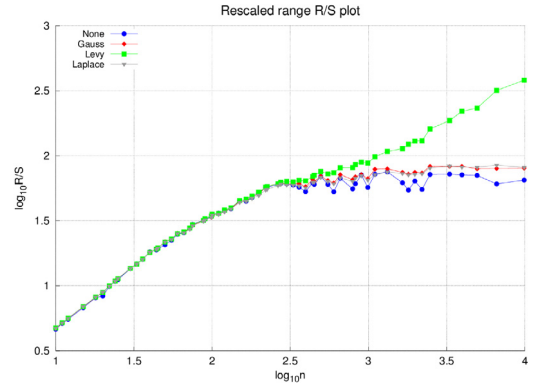


Fig. 8. R/S plot for aggressive tuning of PID controller.

presented in Table 2. Comparison of the plots and obtained Hurst exponent values enables to formulate the following observations:

1. All plots share the same character with two memory scales characterized by different Hurst exponents separated with crossover point n^{cross} . None of the plots may be well approximated with single line (Hurst exponent denoted as $H^{(0)}$). We may clearly distinguish short memory scale $H^{(1)}$ followed by smaller in value long memory scale $H^{(2)}$. It shows that single Hurst exponent $H^{(0)}$ is not good representative for loop dynamics. Thus none of methods for its calculation, like Periodogram [54], Boxed Periodogram [55], Aggregated Variance [56], Detrended Fluctuation Analysis [57], Absolute Value Method [58] or Differential Variance [59] will not

Table 2
PID controllers tuning.

DIST	Tuning	$H^{(0)}$	n^{cross}	$H^{(1)}$	$H^{(2)}$
None	optimal	0.423	198	0.437	0.023
	sluggish	0.467	198	0.461	0.043
	aggressive	0.389	198	0.403	0.021
Gauss	optimal	0.480	198	0.434	0.078
	sluggish	0.519	198	0.460	0.092
	aggressive	0.423	198	0.401	0.053
Levy	optimal	0.584	198	0.425	0.181
	sluggish	0.661	198	0.442	0.238
	aggressive	0.575	198	0.374	0.215
Laplace	optimal	0.484	198	0.433	0.082
	sluggish	0.518	198	0.461	0.090
	aggressive	0.420	198	0.393	0.057

help in solve this problem. Crossover effect disturbs proper application of these algorithms.

2. Crossover point seems to be constant and it does not change despite different controller tuning nor disturbance character. Its value is equal to ~ 200 samples, which in our case reflects 100 [s]. We see that it is exactly the period of setpoint changes. This observation reveals meaning of the crossover point. Short memory exponent preserves information about internal loop dynamic behavior, while second exponent (after crossover point) informs about long memory loop characteristics. Thus, it seems that short-memory scale $H^{(1)}$ is persistence measure that reflects dynamic loop quality assessment.
3. There are significant variations in value of the long memory exponent $H^{(2)}$ depending on disturbance character. They affect long term memory effect, especially disturbances heaving α -stable properties. This is additional effect disqualifying this persistence measure. $H^{(2)}$ is not robust against disturbance character.
4. On the other hand, we see that short memory index keeps its value despite disturbances. Simultaneously, we notice that this relation is invariant against disturbances. Exponent value for aggressive controller is the smallest, while the one for sluggish control is the largest, i.e. $H_{\text{agg}}^{(1)} < H_{\text{opt}}^{(1)} < H_{\text{slug}}^{(1)}$. This result is consistent with other observations about Hurst exponent. The authors have observed that optimal value for Hurst exponent is 0.5, while smaller values denote aggressive control and bigger sluggish one. These features are independent on disturbances. The only difference is that in presented example optimal value is $0.44 \div 0.46$. Thus, $H^{(1)} = 0.5$ does not have to mean optimality. From the control engineering perspective optimal controller tuning depends on current, specific requirements and it may vary with different control tasks. However, its fluctuations according to the response character are permanent.

3.3. Assessment with strong disturbances

The indexes α , γ , IAE and $H^{(1)}$ have qualified for robustness analysis as the result of prior investigations. Assessment ability may be analyzed through simulations of each disturbance scenario for different PID tuning. Time derivative will be fixed at optimal value of $T_d = 2.861$ for all cases. The other two parameters are being changed. Proportional gain varies $k_p = 0.20 \dots 2.50$ with decrement of 0.1, while integration time constant in $T_i = 1.0 \dots 40.00$ changing every 1.0. Altogether 960 simulations of different PID settings are done for each disturbance variant (four scenarios). Experiment results are analyzed and compared through observation of contour plots for obtained relations.

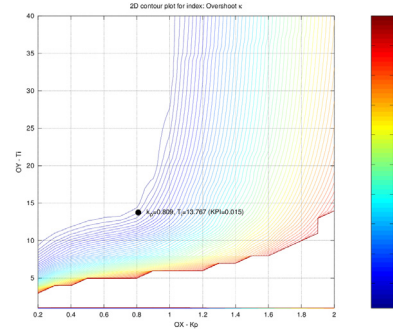


Fig. 9. Surface for overshoot κ .

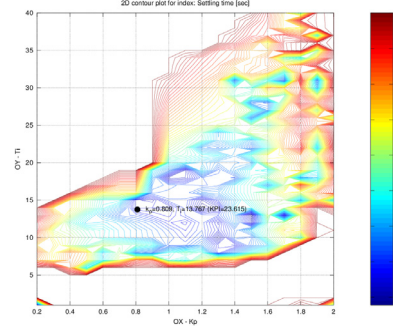


Fig. 10. Surface for settling time T_{set} .

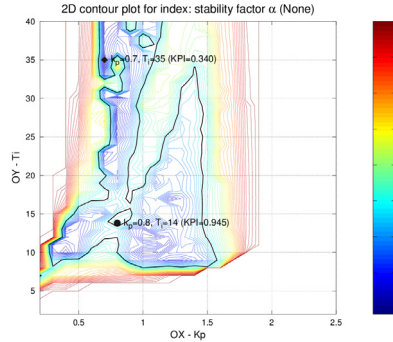
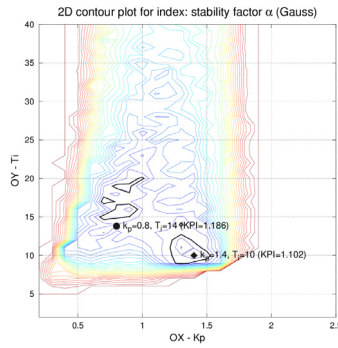
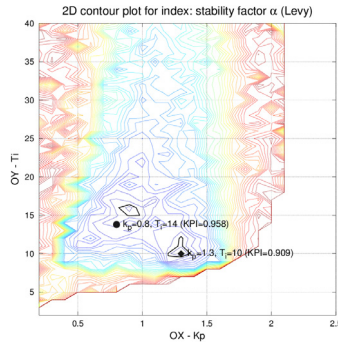
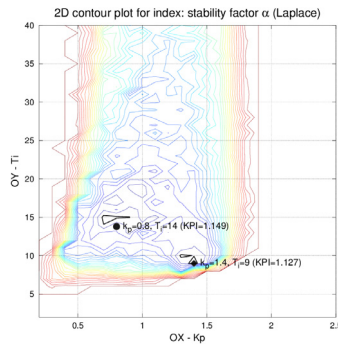
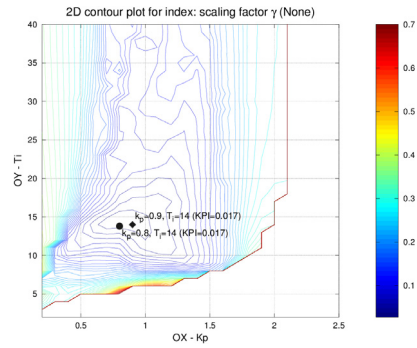
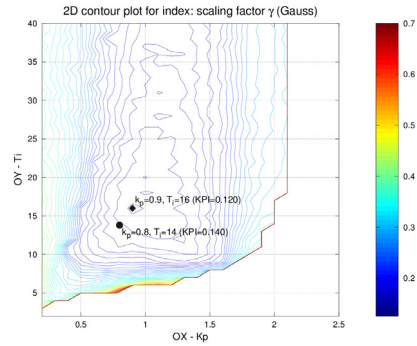
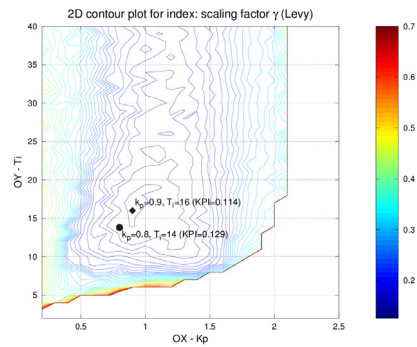
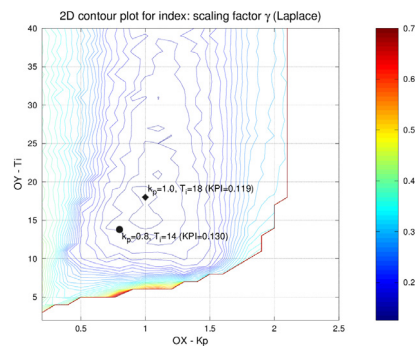


Fig. 11. Index α : no disturbance.

At first, two surfaces are shown. They represent reference indexes of overshoot (Fig. 9) and settling time (Fig. 10) measured for pure step response. These indexes have opposite meaning and good tuning is form of a compromise. Optimal PID setting is shown for reference in both plots. One may see that the surface is in form of single smooth slope from perspective of overshoot with optimal solutions lying on the border of zero overshoot bottom. In contrary, settling time is represented by relatively narrow valley limited with large values of sluggish or oscillating control.

The main experiment drawings are presented in four plot groups. Each of them represents different performance index. There are four separate subplots in each one. They show contour plot for four different disturbance scenarios: none, Gauss, α -stable and Laplace. Two characteristic points are highlighted on each contour plot: minimum value of the index denoted by diamond $\diamond$ and optimal settings marked with circle $\circ$.

The surfaces for the stability factor α are presented in the following figures. Fig. 11 shows non disturbed case, Fig. 12 the surface for the Gaussian disturbance, Fig. 13 α -stable and Fig. 14 Laplace. We notice that the surface is not smooth, although the

Fig. 12. Index α : normal disturbance.Fig. 13. Index α : α -stable disturbance.Fig. 14. Index α : Laplace disturbance.Fig. 15. Index γ : no disturbance.Fig. 16. Index γ : normal disturbance.Fig. 17. Index γ : α -stable disturbance.Fig. 18. Index γ : Laplace disturbance.

shape is unchanged despite disturbance character. Moreover, we see that detected minimum value of the contour is almost the same in case of disturbances affecting the loop. It has more aggressive character with larger proportional gain and smaller integral time constant. In case of no disturbances, minimum point is situated in an opposite, sluggish part with smaller gain and much bigger integral part.

Nonetheless, parameter α of Lévy stable distribution requires more attention. There is a hypothesis that the relation $H = 1/\alpha$ is true [60]. However, we see that it does not hold in our case. Different long-range correlations of the small and large fluctuations may be one of the reasons. On the other hand, if we consider that optimal value of Hurst exponent is neither minimum, nor maximum but ~ 0.5 (or any other value close to it), then also optimal value of stability factor α should be like that. To reflect it, there is also highlighted contour level equal to that of optimal point. Unfortunately, these plots are not very informative. It is very interesting to see how behavior of Hurst exponent looks like.

The surfaces for the scaling factor γ are presented in the following figures. Fig. 15 shows non disturbed case, Fig. 16 the surface

for the Gaussian disturbance, Fig. 17 α -stable and Fig. 18 Laplace. This parameter has similar meaning as normal standard deviation. It represents signal variability considering fat-tail properties. The

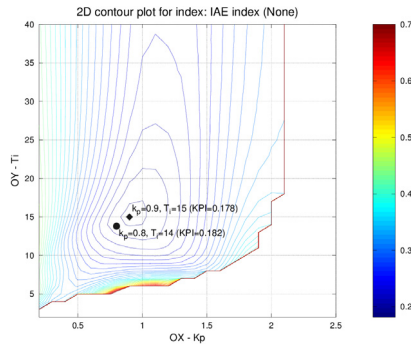


Fig. 19. Index IAE: no disturbance.

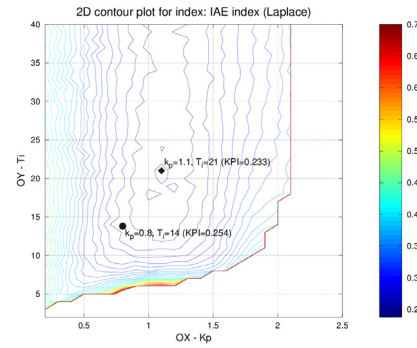


Fig. 22. Index IAE: Laplace disturbance.

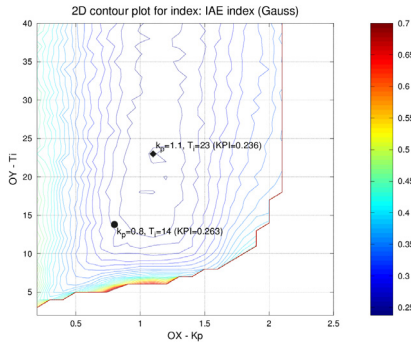
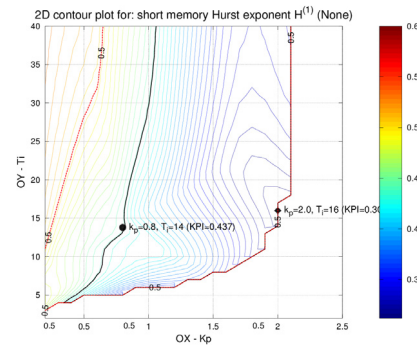
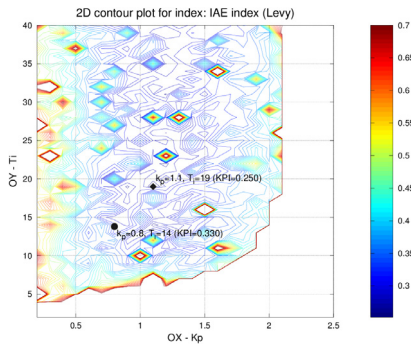
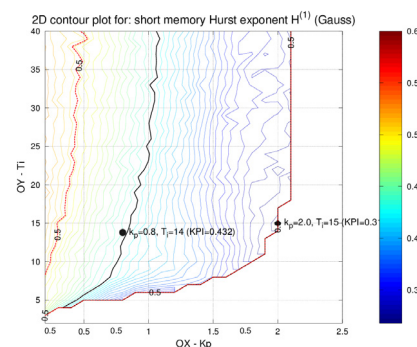
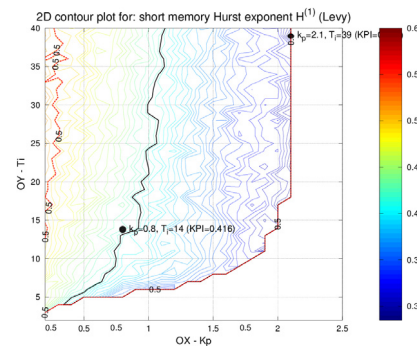


Fig. 20. Index IAE: normal disturbance.

Fig. 23. $H^{(1)}$: no disturbance.Fig. 21. Index IAE: α -stable disturbance.Fig. 24. $H^{(1)}$: normal disturbance.Fig. 25. $H^{(1)}$: α -stable disturbance.

results are promising. The surfaces are smooth and disturbance screening effect is not noticeable. Moreover, surface minimum lies in close proximity to the optimal PID settings. Assessment with scaling parameter γ seems to be robust and effective.

The surfaces for the IAE index are presented in the following figures. Fig. 19 showing non disturbed case, Fig. 20 the surface for the Gaussian disturbance, Fig. 21 α -stable and Fig. 22 Laplace. We notice significant effect of disturbance shadowing in case of fat-tail properties. The plot is highly ragged with rather random position of minimum point. We also notice another interesting behavior. Undisturbed optimal solution is relatively close to the optimal one, while detected best tuning in any of disturbed situations is shifted towards more sluggish control. Solutions are very close to each other despite disturbance character.

Finally the surfaces for the short memory Hurst exponent $H^{(1)}$ are presented in the following figures. Fig. 23 showing non disturbed case, Fig. 24 the surface for the Gaussian disturbance, Fig. 25 α -stable and Fig. 26 Laplace. We see that surfaces maintain different properties. We do not have single valley (nor hill) with an

extremum. Minimum value is situated on the right border, which is edge of stability. This observation confirms hypothesis that minimum Hurst exponent value is not the optimum. We also see that

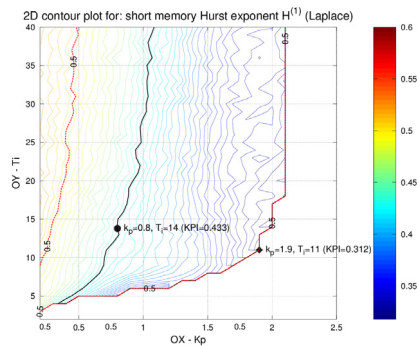


Fig. 26. $H^{(1)}$: Laplace disturbance.

surfaces are relatively smooth, despite disturbance character. It means that properties of the in-loop noises are not shadowing detection abilities with Hurst exponent. It is very important, as such robustness is rare.

Two curves are highlighted on the plots. One shows level of optimal point, i.e. $H^{(1)} \approx 0.4$ and theoretically the best (stochastically uncorrelated process) value $H^{(1)} = 0.5$. Interestingly, each of this curves depicts family of solutions, which is relatively invariant against the disturbance character. Furthermore, the shape of the contour levels resembles the shape of overshoot surface (Fig. 9). Contour level for optimal controller is very close to the one of 1.5% overshoot. This observation has practical consequences. Overshoot may be calculated only from step response in case of small noise-to-signal ratio. Hurst exponent derived from R/S plot is calculated using control error trend considered to be a time series. Such calculations may be done in any situation with no need to perform any dedicated experiment. This is practical facilitation.

We notice that increase in $H^{(1)}$ decreases the overshoot κ . Its optimal value does not have to match the one for uncorrelated process equal to 0.5.

4. Conclusions and further research

Control system quality assessment using fractal persistence measures has been presented and discussed. Simulation experiment has been designed and performed. Relatively simple linear SISO control loop with PID controller has been simulated. The experiment has been disturbed with signals having different statistical properties. Apart from popular Gaussian stochastic process, Laplace and fat-tail Lévy α -stable PDF have been applied. Analyses of industrial data show that majority of loop variables is well explained by the fat-tail distributions. It was confirmed in performed simulations that stable distribution is best fitted to simulated control error. It enables to select stability and scaling factors of Lévy distribution as potentially good quality measures.

Persistence character of control error was also discussed using rescaled range R/S fractal analysis. It has been shown that there are two distinct memory scales: short $H^{(1)}$ and long $H^{(2)}$ separated with crossover point $n^{(cross)}$. It has been shown that analysis with single memory scale (one Hurst exponent $H^{(0)}$) is not appropriate whatever its estimation method is chosen. Evaluation concludes with the selection of short range memory exponent $H^{(1)}$ as a potential candidate for quality measure. Selected parameters have been compared with common IAE index. Main observations are as follows:

1. Loop quality measures strongly depend on properties of loop embedded disturbance. Applied indexes should be based on properly identified PDFs. Analysis of the control error and its histogram helps. Fitting of the distribution function to histogram is one of the possible verification methods.

2. Disturbance shadowing effect appears with Gaussian approach and is still visible for IAE index. Thus, the control engineer should be cautious with the selection, in situations when significant non-Gaussian behavior is visible in analyzed data. Detailed process disturbances review is always needed.
3. Gaussian approach seems to be a good selection in case of no disturbances or while the loop shares such properties.
4. Analysis of integral index IAE shows that it is more biased towards fat-tail distributions scaling factor selecting tuning associated with lower overshoots.
5. Crossover point $n^{(cross)}$ reflects long term fluctuations that occur outside of the loop and are connected with external operating conditions like setpoint variations character.
6. Short memory Hurst exponent $H^{(1)}$ is good and robust index. It reflects dynamical loop properties similar to the overshoot enabling calculation from control error time trend.
7. It is not always true that optimal value for the Hurst exponent is equal to $H^{(1)} = 0.5$. In fact, it depends on actual loop response requirements, loops structure, system complexity (nonlinearity, cross correlations) and disturbance character. Its value is specific depending on the case.

There are many different methods, approaches and measures proposed and used practically in the task of the Control Performance Assessment. Each of them has its applicability scope and drawbacks. The same applies to the proposed fractal approach using the Hurst exponent. First of all the method is dependent on the data used for the assessment. Persistence and fractal analysis may be affected by the data sampling period, especially once we use the shortest memory exponent for dynamical assessment. In such case the crossover point may be small and too large sampling may decrease number of data used for the short Hurst exponent.

The next limiting factor is associated with the need to find appropriate probabilistic distribution function characterizing control error signal. The evaluation has to be performed properly, with large enough number of data, proper bin size and the same as previously small enough sampling period.

Nonetheless, cautious and strict use may enable to extend standard control performance assessment methods revealing normally unavailable loop properties.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- [1] Domański PD, Golonka S, Jankowski R, Kalbarczyk P, Moszowski B. Control rehabilitation impact on production efficiency of ammonia synthesis installation. *Ind Eng Chem Res* 2016;55(39):10366–76.
- [2] Ordys A, Udwehi D, Johnson M. *Process control performance assessment - from theory to implementation*. London: Springer-Verlag; 2007.
- [3] Bauer M, Horch A, Xie L, Jelali M, Thornhill N. The current state of control loop performance monitoring a survey of application in industry. *J Process Control* 2016;38:1–10.
- [4] Domański PD. Non-gaussian properties of the real industrial control error in SISO loops. In: *Proceedings of the 19th International conference on system theory, control and computing*. 2015, p. 877–82.
- [5] Jelali M. *Control performance management in industrial automation: Assessment, diagnosis and improvement of control loop performance*. London: Springer-Verlag; 2013.
- [6] Kantelhardt JW, Zschiegner SA, Stanley HE. Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A* 2002;316:87–114.
- [7] Bardet JM, Bertrand P. Identification of the multiscale fractional brownian motion with biomechanical applications. *J Time Series Anal* 2007;28(1):1–52.

- [8] Zhang J, Jiang M, Chen J. Minimum entropy-based performance assessment of feedback control loops subjected to non-gaussian disturbances. *J Process Control* 2015;24(11):1660–70.
- [9] Shang C, Huang B, Yang F, Huang D. Slow feature analysis for monitoring and diagnosis of control performance. *J Process Control* 2016;39:21–34.
- [10] He F, Xu J. A novel process monitoring and fault detection approach based on statistics locality preserving projections. *J Process Control* 2016;37:46–57.
- [11] Lopes R, Betrouni N. Fractal and multifractal analysis: A review. *Med Image Anal* 2009;13(4):634–49.
- [12] Srinivasan B, Spinner T, Rengaswamy R. Control loop performance assessment using detrended fluctuation analysis (dfa). *Automatica* 2012;48(7):1359–63.
- [13] Spinner T. Performance assessment of multivariate control systems (Ph.D. thesis), Texas Tech University; 2014, p. 1–188.
- [14] Pillay N, Govender P. A data driven approach to performance assessment of PID controllers for setpoint tracking. *Procedia Eng* 2014;69:1130–7.
- [15] Astrom KJ. Computer control of a paper machine—an application of linear stochastic control theory. *IBM J* 1967;11:389–405.
- [16] DeVries W, Wu S. Evaluation of process control effectiveness and diagnosis of variation in paper basis weight via multivariate time-series analysis. *IEEE Trans Automat Control* 1978;23:702–8.
- [17] Toffo F. A methodology to assess the economic returns of advanced control projects. In: *Proceedings of the 1983 American control conference*. 1983, p. 1141–6.
- [18] Harris T. Assessment of closed loop performance. *Canad J Chem Eng* 1989;67:856–61.
- [19] Desborough L, Harris T. Performance assessment measures for univariate feedback control. *Can J Chem Eng* 1992;70(6):1186–97.
- [20] Desborough L, Harris T. Performance assessment measures for univariate feedforward/feedback control. *Can J Chem Eng* 1993;71(4):605–16.
- [21] Tyler ML, Morari M. Performance assessment for unstable and nonminimum-phase systems. *IFAC Proceed Volumes* 1995;28(12):187–92.
- [22] Huang B, Shah SL, Kwok EK. On-line control performance monitoring of MIMO processes. In: *Proceedings of the 1995 American control conference*, vol. 2. 1995, p. 1250–4 vol.2.
- [23] Perrier M, Roche AA. Towards mill-wide evaluation of control loop performance. In: *Proceedings of the control systems*. 1992, p. 205–9.
- [24] Ko B-S, Edgar TF. Performance assessment of cascade control loops. *AIChE J* 2000;46(2):281–91.
- [25] Thornhill N, Huang B, Shah S. Controller performance assessment in set point tracking and regulatory control. *Internat J Adapt Control Signal Process* 2003;17(7–9):709–27.
- [26] Li Z, Evans R. Minimum-variance control of linear time-varying systems. *Automatica* 1997;33(8):1531–7.
- [27] Swanda AP, Seborg DE. Controller performance assessment based on setpoint response data. In: *Proceedings of the 1999 American control conference*, vol. 6. 1999, p. 3863–7 vol.6.
- [28] Shinskey FG. How good are our controllers in absolute performance and robustness?. *Meas Control* 1990;23(4):114–21.
- [29] Spinner T, Srinivasan B, Rengaswamy R. Data-based automated diagnosis and iterative retuning of proportional-integral (pi) controllers. *Control Eng Pract* 2014;29:23–41.
- [30] Harris T, Seppala C, Desborough L. A review of performance monitoring and assessment techniques for univariate and multivariate control systems. *J Process Control* 1999;9(1):1–17.
- [31] Qin SJ. Control performance monitoring a review and assessment. *Comput Chem Eng* 1998;23(2):173–86.
- [32] Meng QW, Gu JQ, Zhong ZF, Ch S, Niu YG. Control performance assessment and improvement with a new performance index. In: *2013 25th Chinese control and decision conference (CCDC)*. 2013, p. 4081–4.
- [33] Zhong L. Defect distribution model validation and effective process control. In: *Proceedings of the SPIE*, vol. 5041. 2003, p. 5041 – 5041 – 8.
- [34] Choudhury MAAS, Shoukat A, Shah SL, Thornhill NF. Diagnosis of poor control-loop performance using higher-order statistics. *Automatica* 2004;40(10):1719–28.
- [35] Liu X, Xie L, Kruger U, Littler T, Wang S. Statistical-based monitoring of multivariate non-gaussian systems. *AIChE J* 2008;54(9):2379–91.
- [36] Domański PD, Ławryńczuk M. Assessment of the GPC control quality using non-gaussian statistical measures. *Int J Appl Math Comput Sci* 2017;27(2):291–307.
- [37] Choi SW, Park JH, Lee I-B. Process monitoring using a gaussian mixture model via principal component analysis and discriminant analysis. *Comput Chem Eng* 2004;28(8):1377–87.
- [38] Yu J, Qin SJ. Multimode process monitoring with Bayesian inference-based finite gaussian mixture models. *AIChE J* 2008;54(7):1811–29.
- [39] Vishnubhotla A. Frequency and time-domain techniques for control loop performance assessment (Ph.D. thesis), University of Alberta, Department of Chemical and Material Engineering, Canada; 1997, p. 1–105.
- [40] Schlegel M, Skarda R, Cech M. Running discrete fourier transform and its applications in control loop performance assessment. In: *2013 international conference on process control (PC)*. 2013, p. 113–8.
- [41] Tepljakov A, Petlenkov E, Belikov J. A flexible MATLAB tool for optimal fractional-order PID controller design subject to specifications. In: *2012 31st Chinese control conference (CCC)*. 2012.
- [42] Alagoz BB, Tan N, Deniz FN, Keles C. Implicit disturbance rejection performance analysis of closed loop control systems according to communication channel limitations. *IET Control Theory Appl* 2015;9(17):2522–31.
- [43] Nesić Z, Dumont GA, Davies MS, Brewster D. CD Control diagnostics using a wavelet toolbox. In: *Proceedings CD symposium, IMEKO*, vol. XB. 1997, p. 120–5.
- [44] Lynch CB, Dumont GA. Control loop performance monitoring. *IEEE Trans Control Syst Technol* 1996;4(2):185–92.
- [45] Yuan H. Process analysis and performance assessment for sheet forming processes (Ph.D. thesis), Queens University, Kingston, Ontario, Canada; 2015, p. 1–214.
- [46] Zhou YF, Wan F. A neural network approach to control performance assessment. *Int J Intell Comput Cybern* 2008;1:617–33.
- [47] Pillay N. Multi-class SVMs for automatic detection and diagnosis in process control loops. *J Control Eng Appl Inform* 2017;19(3):3–12.
- [48] Domański PD. Non-gaussian and persistence measures for control loop quality assessment. *Chaos* 2016;26(4):043105.
- [49] Domański PD. Multifractal properties of process control variables. *Int J Bifur Chaos* 2017;27(6):1750094.
- [50] Godoy JL, Ferramosca A, Gonzalez AH. Economic performance assessment and monitoring in LP-DMC type controller applications. *J Process Control* 2017;57:26–37.
- [51] Seborg DE, Mellichamp DA, Edgar TF, Doyle FJ. *Process dynamics and control*, 3rd edition. Hoboken, NJ: Wiley; 2011.
- [52] Shinskey FG. *Process control: As taught vs as practiced*. Ind Eng Chem Res 2002;41:3745–50.
- [53] Domański PD, Ławryńczuk M. Assessment of predictive control performance using fractal measures. *Nonlinear Dynam* 2017;89:773–90.
- [54] Geweke J, Porter-Hudak S. The estimation and application of long memory time series models. *J Time Series Anal* 1983;4:221–38.
- [55] Hassler U. Regression of spectral estimators with fractionally integrated time series. *J Time Ser Anal* 1993;14(4):369–80.
- [56] Beran J. *Statistics for long-memory processes*, 1st edn., Boca Raton: CRC Press; 1994.
- [57] Peng CK, Buldyrev SV, Havlin S, Simons M, Stanley HE, Goldberger AL. Mosaic organization of DNA nucleotides. *Phys Rev E* 1994;49(2):1685–9.
- [58] Taqqu M, Teverovsky V, Willinger W. Estimators for long-range dependence: an empirical study. *Fractals* 1995;03(04):785–98.
- [59] Teverovsky V, Taqqu MS. Testing for long-range dependence in the presence of shifting means or a slowly declining trend using a variance type estimator. *J Time Ser Anal* 1997;18:279–304.
- [60] Peters EE. *Chaos and order in the Capital Markets: A new view of cycles, prices, and market volatility*, 2nd edition. New York: John Wiley & Sons, Inc.; 1996.