

Development of a High-Efficiency Pitch/Roll Inertial Measurement Unit Based on a Low-Cost Accelerometer and Gyroscope Sensors

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Abstract—This paper describes an original pitch/roll IMU (Inertial Measurement Unit) for Unmanned Aerial Vehicles (UAV) and general robotics applications. The development process is reported and the obtained results of experiments are discussed. The IMU has two key advantages: a) high quality output data is produced at high frequency (1 kHz), b) low cost accelerometer and gyroscope sensors are used. A dedicated 3-axis gimbal has been built to calibrate and test the sensors and later to tune the parameters of the Kalman Filter.

I. INTRODUCTION

The IMU device typically produces attitude data (expressed as Euler angles or a quaternion [1], [2]) at constant output frequency. Attitude data is used in variety of applications such as air-planes [3], mobile robots, drones [4] etc. It is important to emphasize the fact that the higher the data frequency the better, especially for Unmanned Aerial Vehicles (UAVs) such as multi-copter drones because a control loop with high-frequency of intervention makes it possible to obtain good quality of process stabilisation and control. In addition to high frequency, data quality is important as well: high accuracy and good repeatability of measurements is needed.

Easily accessible low-cost gyroscope and accelerometer sensors (which are essential for pitch/roll attitude calculation) give noisy and temperature-drifted raw measurements, better raw measurements are possible when quite expensive sensors are used. The objective of this work is to describe an original pitch/roll IMU for UAVs and general robotics applications. Development and tuning of the IMU are described. The discussed IMU is able to generate high quality output data at reasonably high frequency (1 kHz) in spite of the fact that low cost accelerometer and gyroscope sensors are used. This work also describes a dedicated 3-axis gimbal which has been built to calibrate and test the sensors and later to tune the parameters of the Kalman Filter. The results of experiments are reported and discussed. The built IMU can compete with expensive off-the-shelf IMUs, e.g. [5], [6].

There are many different methods of IMU calibration. In [7] the sensors are calibrated in static conditions using precisely milled aluminium form. Placing the IMU in a series of well-known orientations (in the form) helps reduce the accelerometer and magnetometer errors. Sensors calibration in dynamic conditions is described in [8] where IMU is being

turned simultaneously over all 3 axes. That approach also can be used to reduce accelerometer and gyroscope errors. Static calibration does not measure subtle inaccuracy of the accelerometer over the full-scale of sensor data. The 360°-rotation of the IMU is much better for mapping sensor non-linearities, but the whole calibration becomes a much more complex task. A simplified approach, i.e. single axis at the time, has been used in this research.

II. PITCH/ROLL INERTIAL MEASUREMENT UNIT

The pitch/roll Inertial Measurement Unit which computes attitude angles is schematically depicted in in Fig. 1, where φ_x is the searched tilt angle, ω_x is the angular velocity around IMU center, $\vec{g}$ is a dummy gravity vector ($|\vec{g}| = g$, where g is the Earth gravity acceleration constant), $\vec{a}_x$ and $\vec{a}_z$ are the gravity vector components, measured by the accelerometer along X and Z axis respectively. The considered IMU can be treated as two independent single-axis IMUs for pitch and roll tilt angles, respectively. This assumption is correct if there are no cross-axis couplings or they are small enough and may be neglected. All further calculations are based on that assumption and are performed for a single axis only (X axis assumed to be the roll value).

There are two sensors available in the considered IMU: an accelerometer and a gyroscope. The 3-axis accelerometer measures dynamic accelerations (movement) and a static acceleration which is equal to the gravity vector $\vec{g}$. The total acceleration measured by the sensor consists of three vector

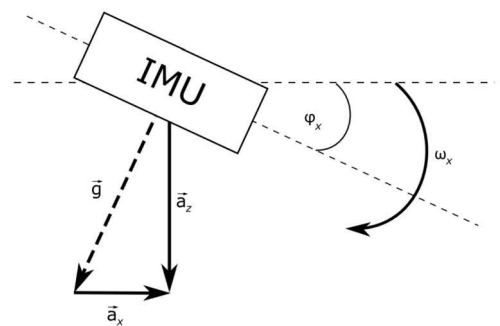


Fig. 1: Inertial Measurement Unit (IMU): values for a single axis

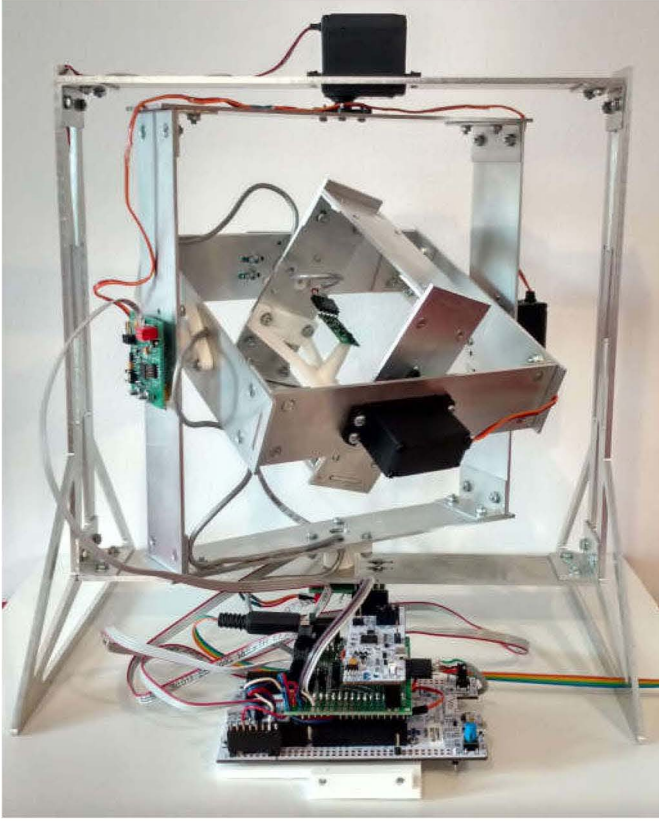


Fig. 2: Automated 3D gimbal

coordinates $[a_x \ a_y \ a_z]$. Values are scaled in $g = |\vec{g}|$. In ideally static conditions one may calculate roll and pitch angles (in general: any tilt angle) using the accelerometer only. If the IMU is mounted in a moving device, such as a drone, accelerometer readings are highly affected by dynamic accelerations. Therefore, in the IMU a gyroscope sensor is used as well. The 3-axis gyroscope measures angular speed rate around each axis (expressed in $^\circ/\text{s}$). In theory, integrating the angular speed over time would also give the tilt angle value. However in practice even low-density noise of the gyroscope, integrated many times, leads to a significant error of the tilt angle calculation. Additionally, silicon gyroscopes are affected by a temperature drift of the steady-state value.

III. 3-AXIS GIMBAL

A dedicated 3-axis gimbal, shown in Fig. 2, has been built to measure IMU sensors' features and to precisely compare IMU output values with real absolute attitude angles. It consists of three aluminium square frames and the sensors board mounted exactly in the pivot point (in the middle of the smallest frame). Each frame can rotate in different axis: X , Y and Z . There is a continuous rotation servo mounted on one side of each axis, while on the other spindle there is a contactless magnetic encoder (absolute position, ca. 0.1° resolution). Each servo is controlled by a cascade PID controller (implemented in a STM32F0-family microcontroller). The first PID keeps the desired angular speed of the servo (using encoder data changes

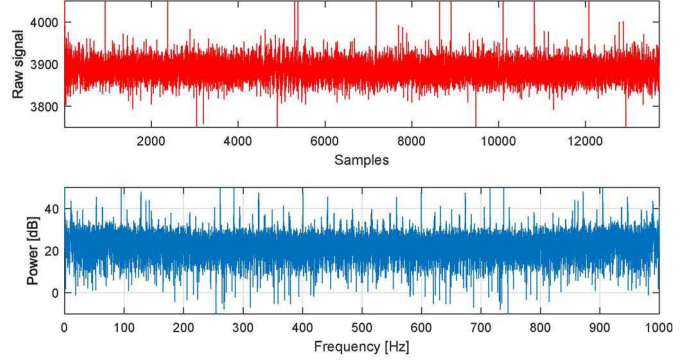


Fig. 3: Accelerometer signal noise

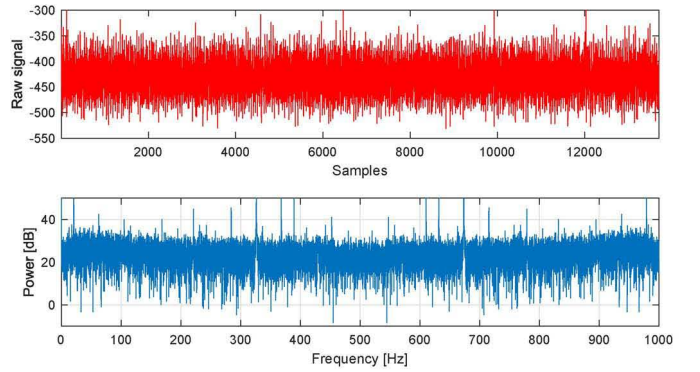


Fig. 4: Gyroscope signal noise

as a feedback) and the second PID keeps watching the absolute tilt angle (again, using the encoder input) and adjusts the desired speed of the first PID. The whole control loop works at 50 Hz, and the encoder reads the data asynchronously as fast as it can (approx. 1000 samples/s). The controller can hold the desired absolute position of particular frame as well as rotate the frame up to programmed absolute position with constant angular speed. Hence, it is possible to repeat desired trajectories for each axis. That property of the gimbal is very helpful when a number of experiments must be performed.

The following sensors are used in the described configuration:

- IMU: 1x ST LSM6DS33 – 3D accelerometer and 3D gyroscope in a single chip [9],
- Gimbal: 3x ams AS5161 – magnetic angle position sensor (absolute encoder) [10].

The sensor readings are affected by the following issues:

- Accelerometer and gyroscope: signal noise, shown in Figs. 3 and 4, can be treated as a white noise.
- Accelerometer and gyroscope: non-linearities of the signal across the full range of output values. This particular issue is not significant for the sensors used. Hence, it can be neglected in further calculations.
- Gyroscope only: a temperature drift of the steady-state signal level, shown in Fig. 5.

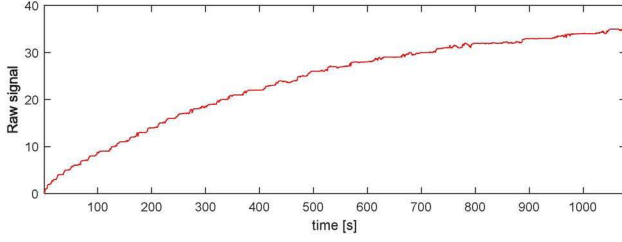


Fig. 5: Temperature drift of gyroscope signal

IV. KALMAN FILTER AS THE IMU SENSOR-FUSION ALGORITHM

There are many sensor fusion algorithms which compensate mentioned sensor issues, eg. [11]–[13]. In this work the Discrete Kalman Filter [14] has been implemented. The objective of the Kalman Filter is to find the tilt angle of the IMU regardless of the sensors' noise and any dynamic accelerations (movements). The idea is to use the gyroscope as the main source of input data, then correct the result and the gyroscope's temperature drift using accelerometer data. The ideal model would be

$$\varphi(t) = \int_{\tau=0}^t c_g \omega(\tau) - \omega_0 d\tau \quad (1)$$

where $\varphi(t)$ is the tilt angle (in degrees) for time t , c_g is the gyroscope scaling factor, which depends on selected sensor sensitivity and data resolution, and ω_0 is the gyroscope bias indicating the output value at a steady-state (angular speed $\omega = 0$). In practice, the ω_0 value is noisy and depends on sensor's temperature, thus has to be corrected as quick as possible to avoid integrating errors over long periods of time. Instead, the following discrete state-space model for Kalman Filter is used

$$\begin{cases} X(k+1) = AX(k) + B(c_g \omega(k) T) \\ y(k) = CX(k) \end{cases} \quad (2)$$

where model parameters are

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (3)$$

$$C = [1 \quad 0], \quad X = \begin{bmatrix} \varphi \\ \omega_0 \end{bmatrix}, \quad T = 0.001 [s]$$

The accelerometer readings are not used in the model itself, but they are used in the correction phase (*a posteriori*) of the Kalman Filter. According to Fig. 1, the tilt angle (in degrees) in ideal conditions may be calculated from

$$\begin{aligned} \varphi(t) &= \arcsin(c_a a_x(t)) (180/\pi) \\ &= \arccos(c_a a_z(t)) (180/\pi) \end{aligned} \quad (4)$$

where $a_x(t)$ is the accelerometer X axis vector component and $a_z(t)$ is the accelerometer Z axis vector component for time t . c_a is the accelerometer scaling factor which depends on selected sensor sensitivity and data resolution. It is necessary to note that Eq. (4) is valid for $-90^\circ \leq \varphi \leq 90^\circ$ range. For tilt angle exceeding that range an additional phase shift and a sign correction is needed.

Taking into account the above remarks, the Kalman Filter calculations for the prediction phase (*a priori*) are:

- 1) Calculate predicted output $\hat{y}(k)$ using the state-space model given by Eq. (2).
- 2) Update covariance matrix P

$$P = (APA^T) + Q \quad (5)$$

Q is the covariance matrix which represents measured gyroscope signal variance (σ_g^2) at the diagonal of the matrix

$$Q = \begin{bmatrix} \sigma_g^2 & 0 \\ 0 & \sigma_g^2 \end{bmatrix} \quad (6)$$

The correction phase (*a posteriori*) of the Kalman Filter consists of following steps:

- 1) Determine accelerometer-based tilt angle (see eq. (4)). It is next used as the correction value z .
- 2) Calculate Kalman Filter gain K

$$K = (PC^T)(CPC^T + R)^{-1} \quad (7)$$

- 3) Correct the state values

$$X = X + K(z - CX) \quad (8)$$

- 4) Correct the covariance matrix P

$$P = (I - KC)P \quad (9)$$

- 5) Calculate the final output value $y(k)$ which is in fact the searched tilt angle $\varphi(k)$

$$y(k) = CX = [1 \quad 0] \begin{bmatrix} \varphi(k) \\ \omega_0(k) \end{bmatrix} = \varphi(k) \quad (10)$$

In Eq. (7) R is the variance of the accelerometer signal (σ_a^2), adjusted by a tuning parameter k_a

$$R = k_a \sigma_a^2 \quad (11)$$

The tuning parameter k_a can be used to make the overall IMU calculations more robust to dynamic accelerations measured by the accelerometer sensor. It means that in fact the final tilt angle is smoothed and depends more on the gyroscope readings than on the accelerometer readings, while the latter is still used to find the gyroscope bias value. After a series of experiments it has been found out that the coefficient k_a has to be within the range $10 \dots 1000$.

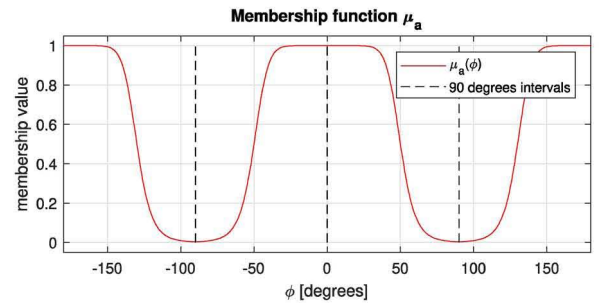


Fig. 6: $\mu_a(\varphi)$ membership function

For the final IMU Sensor-Fusion Algorithm additional improvement has been introduced: the correction tilt angle value z (accelerometer-based) is calculated based on X or Z axis component of the accelerometer which depends on the predicted tilt angle $\hat{y}(k)$. The improvement is important because it increases accuracy of the calculations for tilt angles in the neighbourhood of $90^\circ \pm 15^\circ$ and $-90^\circ \pm 15^\circ$. From Eq. (4) and Fig. 1 one may clearly conclude that both formulas works well only in specified sub-ranges of tilt angles. In order to take advantage from both formulas, a membership function μ_a shown in Fig. 6 is introduced. It is characterised by the following equation

$$\mu_a(\varphi) = \begin{cases} \left(1 + \left|\frac{\varphi}{50}\right|^{10}\right)^{-1}, & \text{if } |\varphi| \leq 90^\circ \\ \left(1 + \left|\frac{180 - \varphi}{50}\right|^{10}\right)^{-1}, & \text{if } \varphi > 90^\circ \\ \left(1 + \left|\frac{-180 - \varphi}{50}\right|^{10}\right)^{-1}, & \text{if } \varphi < -90^\circ \end{cases} \quad (12)$$

The weighting functions w_x and w_z are defined as follows

$$\begin{aligned} w_x &= \mu_a(\varphi) \\ w_z &= 1 - w_x \\ z_x &= \arcsin(c_a a_x(t)) (180/\pi) \\ z_z &= \arccos(c_a a_z(t)) (180/\pi) \end{aligned} \quad (13)$$

and finally the accelerometer-based tilt angle used as the Kalman Filter correction input z is calculated

$$z = w_x z_x + w_z z_z \quad (14)$$

It must be emphasised that if one takes into account pure maths and physics only, the membership function $\mu_a(\varphi)$ should switch exactly every 45° . However, it is defined in a bit different way since the Z axis of the accelerometer is a little bit more noisy and has more non-linearities in comparison to the X and Y axes. After a series of experiments it has been decided that the membership function should be weighted a little bit in favour of the X axis.

V. TEST RESULTS

The following quality metrics are defined for the IMU experiments:

- a) Overall error for predefined gimbal trajectory

$$E = \sqrt{\frac{1}{N} \sum_{i=1}^N (\Psi(i) - \varphi(i))^2} \quad (15)$$

where $\Psi(i)$ is the angle at i -th step in the trajectory

- b) Static accuracy defined as standard deviation of IMU output when there is no movement

$$E_s = \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (\varphi(i) - \mu_\varphi)^2} \quad (16)$$

where

$$\mu_\varphi = \frac{1}{N} \sum_{i=1}^N \varphi(i) \quad (17)$$

- c) Dynamic accuracy E_d defined as a mean of square errors around every maxima and minima of the test gimbal trajectory
- d) E_b defined as a standard deviation of calculated gyroscope bias ω_0

$$E_b = \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (\omega_0(i) - \mu_{\omega_0})^2} \quad (18)$$

where

$$\mu_{\omega_0} = \frac{1}{N} \sum_{i=1}^N \omega_0(i) \quad (19)$$

Experiments have been conducted in a stable room temperature, electronic components have been already warmed-up and each experiment time has been relatively short, so the expected gyroscope bias value should be quite stable.

Four experiments have been conducted: low tilt angle trajectory (ca. $\pm 50^\circ$) and high tilt angle trajectory ($> \pm 90^\circ$) for well-calibrated sensors (using gimbal encoders) and for sensors calibrated based on their datasheets only. Figs. 7 and 10 show overall errors comparison for low- and high-tilt angle experiments. While the static stability E_s is similar (as seen in Fig. 12), the dynamic errors E_d , which are important in practical implementations, are two times smaller for the calibrated IMU experiments (Figs. 8 and 11). Experiments show that the repeatability of output values is very good too. Tuning the k_a parameter from Eq. (11) shows that the IMU can be robust enough to handle dynamic accelerations (movements) as well as higher accelerometer noise, e.g. vibrations of a running motor. Additionally from Figs. 9 and 13 one may easily conclude that fully calibrated sensors give much lower model errors. The gyroscope bias should be almost constant over short time periods (a few seconds) due to the nature of the sensor. Quick changes of the gyroscope bias without a significant change of the ambient temperature means that there are significant input data errors: either the gyroscope or the accelerometer values are wrong. Calibrating the IMU using the gimbal minimizes the gyroscope bias changes. The remaining fluctuations still observed in Figs. 9 and 13 comes from non-linearities of the accelerometer and the gyroscope. Future research work may be focused on mapping those sensor issues as well giving as a result smooth sensor readings over a wide range of physical conditions.

VI. CONCLUSIONS

Due to using the gimbal, equipped with precise absolute encoders, it is possible to achieve outstanding precision, repeatability and overall data quality of the IMU, at very high sampling frequency of 1kHz, using ordinary, low-cost sensors. It is worth to mention that the commercial IMU cost is €500–€2800 while the cost of the gimbal is approx. €150 and cost of the complete sensor board used in this research is just €17.

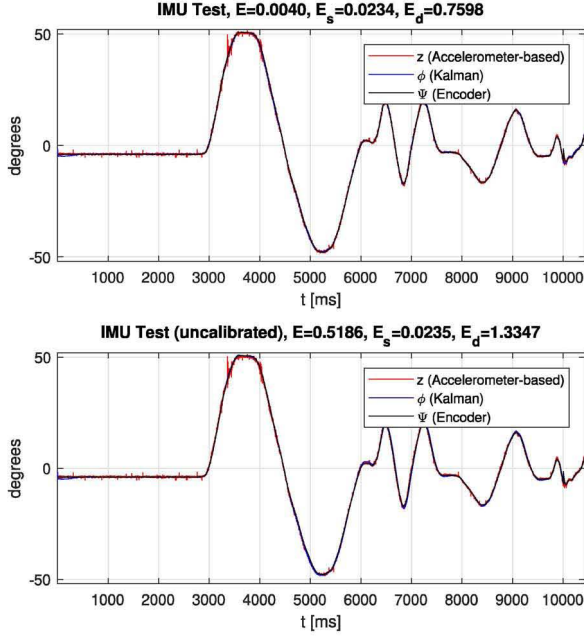


Fig. 7: Low tilt angle test results

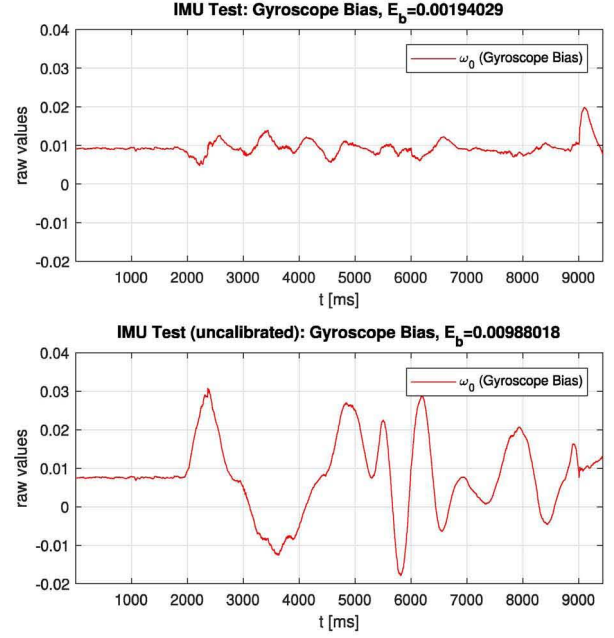


Fig. 9: Low tilt angle test results – gyroscope Bias ω_0

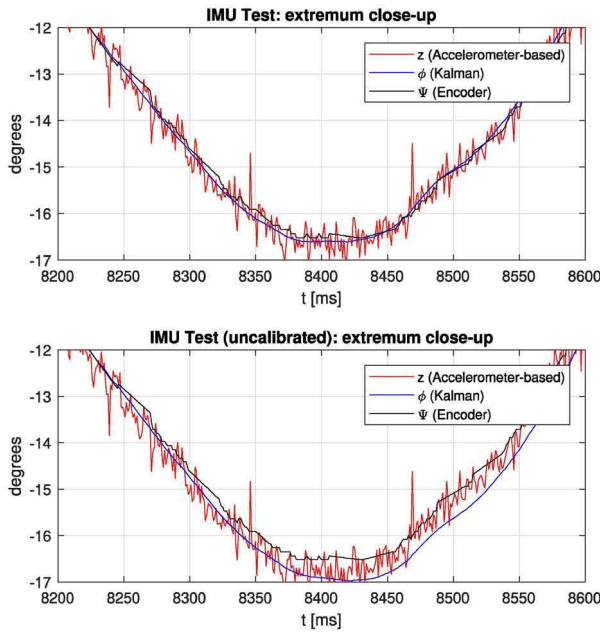


Fig. 8: Low tilt angle test results – close-up

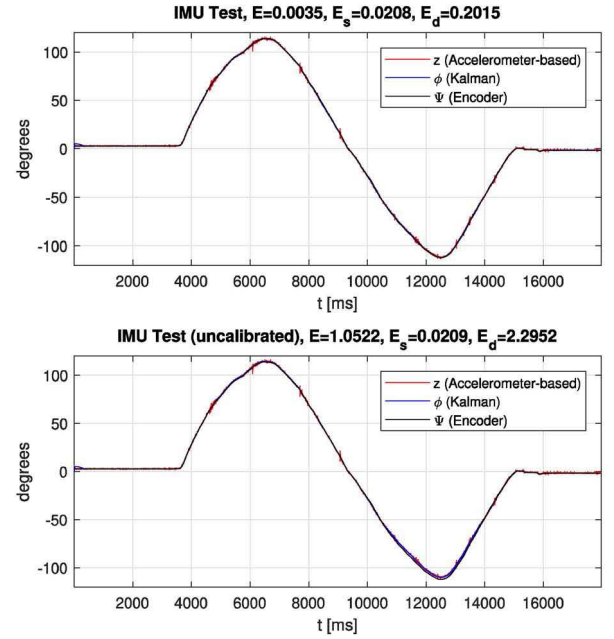


Fig. 10: High tilt angle test results

After a series of experiments all the parameters, i.e. c_a , c_g , k_a have been tuned. As observed from Figs. 7 and 10, low error values (defined by Eqs. (15)-(19)) are obtained. Static and dynamic accuracy, as well as stability, are comparable to those of very expensive industrial-grade IMUs. It is worth emphasising that such a good result has been achieved using just a simple Kalman Filter. High accuracy and high frequency IMU can be used e.g. in all the robotic applications where very fast control loop are necessary like drones or fast mobile robots (generally: most of the UAV applications). Outstanding static

accuracy of the IMU can also eliminate the need of installing absolute encoders to determine e.g. robotic arm attitude.

Taking into account the obtained results, it is possible to formulate the following future research directions and some ideas for further improvements:

- Full calibration and mapping of non-linearities over a wide temperature range, e.g. $-10^{\circ}\text{C}.. +40^{\circ}\text{C}$.
- Checking advantages (if any) of the Extended Kalman Filter vs. standard Kalman Filter for IMU Sensor-Fusion Algorithm.

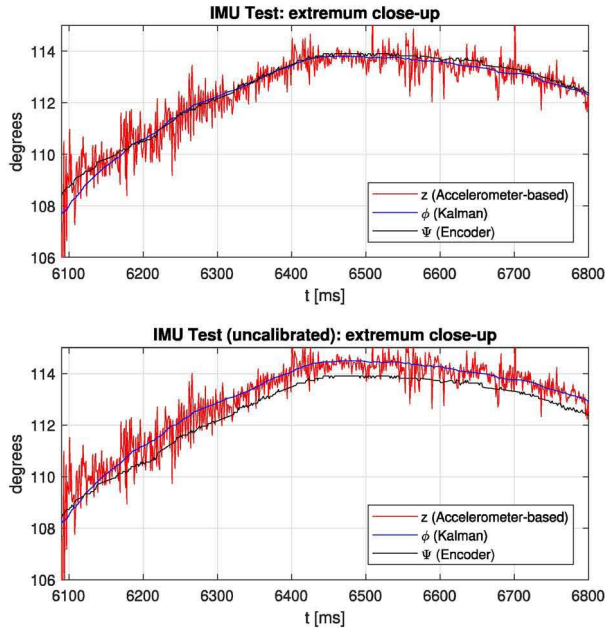


Fig. 11: High tilt angle test results – close-up

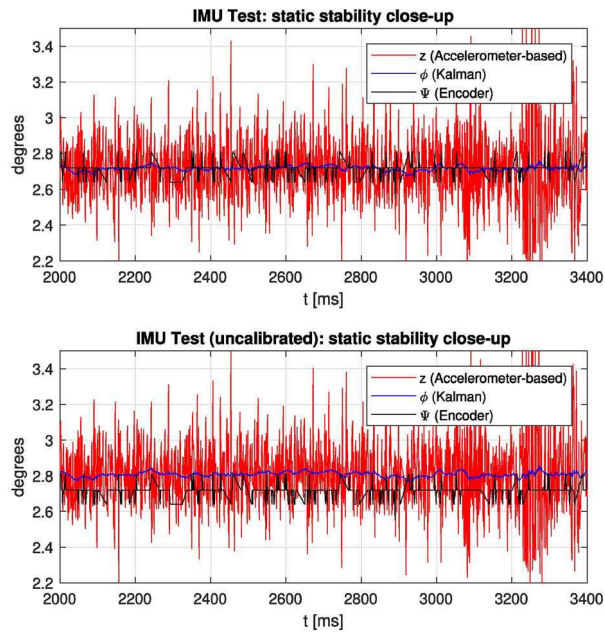


Fig. 12: High tilt angle test results – static stability; for low tilt angle tests results were very similar to this one

- Enabling the third sensor (a 3D magnetometer), its full calibration (including ellipsoid fitting) and usage to implement a full 3D IMU: for pitch, roll and yaw attitude estimation.
- Usage of the magnetometer to improve pitch/roll data quality (depending on current yaw position).

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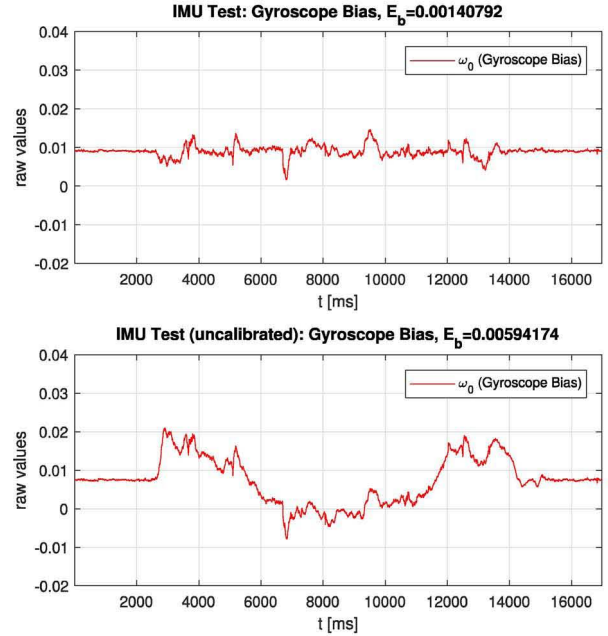


Fig. 13: High tilt angle test results – gyroscope Bias ω_0

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