

Control Performance Assessment of the Disturbance with Fractional Order Dynamics

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Abstract. The fractional order system is the generalization of the conventional integer order control system. The control system subject to the disturbance with fractional order dynamics is discussed in this paper. Error signals from the control system are analyzed by Hurst fitting with crossovers, α -stable distribution and multifractal analysis. The fractional order coefficient is selected as the useful parameter of the long range dependence properties for the control performance assessment.

Introduction

The statistical control quality measures are widely used in the current industry to assess control loops, which are based on the classical Gaussian approach. However, the industrial process presents many non-Gaussian behaviors, such as long range dependency (LRD), self-similarity, power-law of autocorrelation, infinite variance, spiky signals. These phenomena are originated from the fractional calculus. Domański reports that majority has the fat-tailed property after reviewing of data from industrial process, which is due to the complexity, correlation, time varying delays and human interventions [1]. Liu et al. propose a novel control performance assessment (CPA) method with the fractional order signal processing (FOSP) techniques in [2]. In this paper, we consider and extend the control system under the disturbance with fractional-order dynamics with FOSP [3].

Methods and Simulations

In time series analysis, the autoregressive (AR) model is to build the short memory of the previous data and moving average (MA) is to model the random noise. The fractional integrated (FI) is to characterize the long memory properties. The ARFIMA (p, d, q) model of a time series X_t is defined as follows [4]:

$$\left(1 - \sum_{i=1}^p \phi_i(z^{-1})^i\right) (1 - (z^{-1}))^d (X_t - \mu) = \left(1 + \sum_{i=1}^q \theta_i(z^{-1})^i\right) \varepsilon_t, \quad (1)$$

where $\phi_1, \dots, \phi_p$ are AR parameters, $\theta_1, \dots, \theta_q$ are MA parameters, μ is the expectation of X_t , ε_t is the white noise and z^{-1} denotes the unit delay. In contrast to the ordinary ARIMA process, the difference operator d in ARFIMA (p, d, q) is allowed to take non-integer (fractional) values.

LRD, which is characterized by the Hurst exponent, means that there is a strong coupling effect between values at different time separations. In addition, the fractional order difference (integral) coefficient d has a closed relation with Hurst exponent H and characteristic exponent α .

$$d = H - 1/\alpha. \quad (2)$$

Therefore, the fractional order coefficient d is used to characterize the LRD property. In this paper, we build the control system simulation with LRD disturbances by changing the value of d in the fractional order difference filter $(1 - (z^{-1}))^{-d}$ in Figure 1. It should be noted that LRD noise will be reduced to the short memory without any coupling effect when $d = 0$. Controller tuning remains the same in the simulations and 80000 sampled

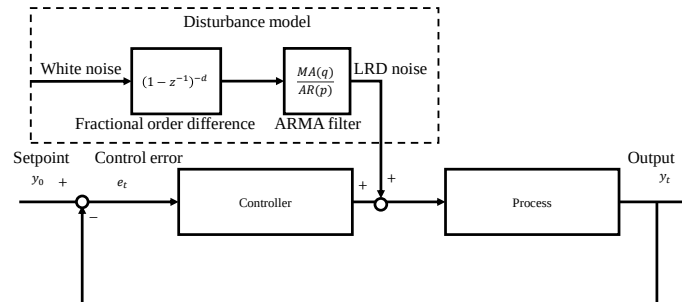


Figure 1: The control system under the fractional-order disturbances.

points are generated. The distribution of error signals are becoming broader with the increasing fractional order coefficient d in Figure 2. Therefore, the disturbance with LRD can be characterized by the ARFIMA model, in which fractional order difference filter d is the indicator of coupling effects. The higher value of d represents

settings for the controller under the LRD disturbance, if we would like to control/tune the system to the same behavior. For example, the ion aggregation phenomenon during the plasma etching process can introduce this type of disturbance in the semiconductor manufacturing. Meanwhile, the repeatability of the run-to-run batch process is the primary concern for the product. Therefore, the detection and evaluation of the fractional order coefficient d should be considered as a desirable parameter in CPA.

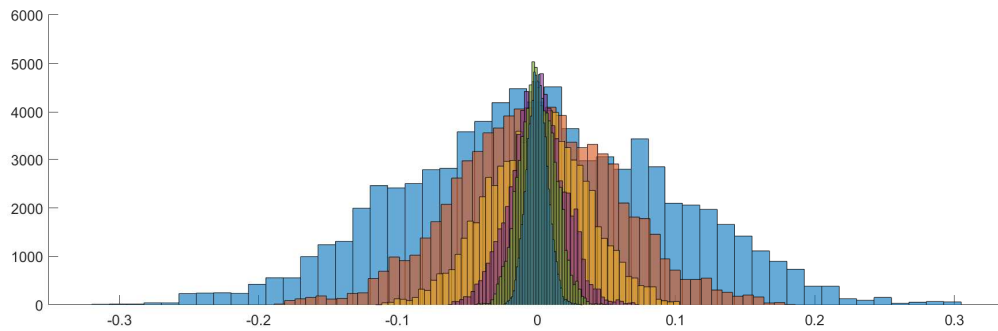


Figure 2: Error histogram with fractional order difference coefficient $d=0.0, 0.1, 0.2, 0.3, 0.4, 0.5$ (from middle to the edge).

Analysis and discussions

First, the Hurst exponent is fitted and estimated based on the error signals. However, original rescaled range (R/S) method is subject to the trend of data. In this section, we propose the double Hurst fitting method at different scales. Second, the classical Gaussian distribution statistical analysis can be extended to the α -stable distribution. The characteristic exponent α describes the tail of the distribution. β is the skewness, δ and γ indicate mean and scale (variance). Particularly, $\alpha = 2, 1, 1/2$ represent the Gaussian, Cauchy, Lévy distribution respectively.

frac-order d	1-Hurst H_1	2-Hurst H_1, H_2	esti-Hurst $\hat{H}$	tail α	skewness β	scale γ	mean δ	spec-width ΔH
0.5	0.825	1.045, 0.604	1.0000	2.0000	1.0000	0.06820	-0.0000	0.494
0.4	0.820	1.053, 0.587	0.9019	1.9923	-0.1313	0.0404	-0.0002	0.356
0.3	0.816	1.062, 0.568	0.8000	2.0000	-1.0000	0.0252	-0.0002	0.292
0.2	0.817	1.059, 0.573	0.7026	1.9895	0.0670	0.0135	-0.0000	0.242
0.1	0.820	1.061, 0.577	0.6000	2.0000	1.0000	0.0090	0.0002	0.148
0.0	0.818	1.073, 0.561	0.5010	1.9962	1.0000	0.0052	0.0001	0.078

Here are some novel discussions from the above results:

- Since the classical R/S Hurst fitting method does not consider the LRD (coupling) property of the time series, it should be modified to the multiple-Hurst fitting algorithm with multiple spline curves.
- For LRD disturbance with larger d values, CPA should choose the multiple-Hurst fitting results in the small scale. For smaller d , large scale Hurst fitting results should be utilized.
- The fractional order difference parameter d has a close relation with the Hurst exponent H with Equation 2.
- The multifractal detrending fluctuation analysis (MFDFA) describes the total set of local Hurst exponents (Hölder exponent), in which the width of the spectrum $\Delta H \approx d$.

Conclusions

The paper extends some approaches to analyze the disturbance with LRD by using the FOSP techniques. The fractional order difference coefficient d is considered and evaluated with Hurst fitting method, α -stable distribution, multifractal analysis. The proposed CPA method for the system with the fractional-order dynamics is helpful and may be applied in various situations.

References

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