

Model Predictive Control and energy optimisation in residential building with electric underfloor heating system

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ABSTRACT

This work is concerned with modelling, Model Predictive Control (MPC) and on-line energy optimisation of a residential building in which an underfloor heating system based on electric heating foils is used. At first, the first-principle model is developed to determine air temperature inside the building. Low-order linear models for MPC are found. Tuning of MPC is discussed, both set-point change and disturbance compensation cases are discussed. Next, two control system structures are considered: MPC with a constant temperature set-point and MPC cooperating with a set-point optimiser which repeatedly calculates on-line the set-point in order to minimise the amount of energy used. Effectiveness of these structures is validated in simulations using a real scenario of outdoor temperature changes. In the first structure a natural method to reduce energy used is to slightly reduce the temperature set-point, e.g. reduction of the set-point from 21 °C to 18.5 °C leads to reducing the energy usage by approx. 1430 KWh during the heating season. Introduction of the set-point optimiser makes it possible to further reduce energy used by approx. 300 KWh annually. The presented solution is computationally efficient since in MPC and set-point optimisation the classical quadratic optimisation method is used on-line.

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1. Introduction

Radiant flow heating is an efficient solution for delivering thermal energy in a short time period. However, due to high operational costs (electrical energy price is significantly higher than the price of natural gas) the use of such systems is limited. The electrical heating systems are of high efficiency and are used mostly as low-temperature heating systems. The typical examples are surface or panel heating. Their efficiency is higher compared to air distribution systems [1] or high temperature radiator systems [2]. Due to the radiation heat transfer, the thermal comfort is sufficient at even lower air temperature than in traditional heating solutions i.e. hydronic radiators [2,3]. Furthermore, the low thermal inertia of electrical flow heating systems allows one to obtain the desired room temperature value in a very short time period. Therefore, it is necessary to optimise electric energy usage. Furthermore, the trend in rapid decrease of PhotoVoltaic (PV) solutions prices and

increased efficiency of energy storage technologies may lead in future to coupling PV panels with electrical floor heating system, making this solution economically reasonable.

Many studies have been made so far on investigation of floor heating systems. Werner-Juszczuk [4] performed an experimental and numerical investigation of lightweight floor heating with metallised polyethylene radiant sheet. The Finite Element Method (FEM) simulation in ANSYS software was conducted to determine temperature and heat flux distribution in inter-floor ceiling model. The heat flux values obtained from measurements were compared with the numerical simulation results and the difference was up to 8.2%. Smaller errors, less than 3%, were obtained when the surface temperatures were compared. Zhao et al. [5] studied a 20 mm thick concrete tile for radiant heating application. They developed an innovative fabrication procedure to set the top and bottom electrodes in a tile construction. They found that the electrical energy consumption was 0.775 kWh/m² which was lower than that of the typically used heating cables. The authors also performed simulation in ANSYS Fluent to simulate the temperature distribution inside the room with the conductive concrete tile heating floor. The results of temperatures shown good agreement with the

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Nomenclature*First-principle model*

A_{1a}	vertical wall area without windows and front door (19.5 m ²)	T_1, T_6, T_7, T_{12}	temperatures of the outer surface of the building envelope corresponding to north, south, east and west directions (°C)
A_{1b}, A_2	vertical wall areas without windows (21.5 m ²)	T_2, T_5, T_8, T_{11}	temperatures of the interface between insulation layer and building wall corresponding to north, south, east and west directions (°C)
A_3	horizontal wall area (100 m ²)	T_3, T_4, T_9, T_{10}	temperatures of the inner surface of the building wall corresponding to north, south, east and west directions (°C)
A_{door}	area of entrance door (2 m ²)	T_{13}	temperature of the inner surface of the building roof (°C)
A_{heat}	heating surface of building (100 m ²)	T_{14}	temperature of the inner surface of the building roof insulation (°C)
A_{wd}	area of windows at the building wall (4.5 m ²)	T_{15}	temperature of the outer surface of the building roof insulation (°C)
c_{air}	specific heat of air (1005 J/(kg K))	T_{16}	temperature of the inner surface of the flooring (°C)
c_c	specific heat of ceiling material (1000 J/(kg K))	T_{17}	temperature of the outer surface of the flooring (°C)
c_f	specific heat of floor layer material (2000 J/(kg K))	T_{18}	temperature of the outer surface of the spout (°C)
c_{ins}	specific heat of wall insulation material (1300 J/(kg K))	T_{19}	temperature of the outer surface of the basement insulation (°C)
$c_{ins,c}$	specific heat of ceiling insulation material (1300 J/(kg K))	T_{20}	air temperature in the room (°C)
$c_{ins,s}$	specific heat of spout insulation material (1300 J/(kg K))	T_{ext}	external temperature (°C)
c_s	specific heat of spout material (1000 J/(kg K))	$T_{ext,g}$	ground temperature (5 °C)
c_{wall}	specific heat of wall material (1000 J/(kg K))	t	continuous time (s)
H	height of the building (2.6 m)	U_{door}	overall heat transfer coefficient of door (1.9 W/(m ² K))
h_{ext}	heat transfer coefficient from the external air (10 W/m ² K)	U_{wd}	overall heat transfer coefficient of window (1.5 W/(m ² K))
$h_{ext,g}$	ground overall heat transfer coefficient (10 W/m ² K)	V_{air}	volume of air in the room (m ³)
$h_{in,c}$	indoor air heat transfer coefficient for ceiling surface (10 W/m ² K)	V_i	volumetric flow rate of air (m ³ /s)
$h_{in,f}$	indoor air heat transfer coefficient for floor surface (6 W/m ² K)	W	width of the building (10 m)
$h_{in,v}$	indoor air heat transfer coefficient for vertical walls (8 W/m ² K)	Δx_c	thickness of ceiling material (0.2 m)
h_{rad}	radiation heat transfer coefficient (W/m ² K)	Δx_f	thickness of floor layer material (0.010 m)
k_c	thermal conductivity of ceiling material (2.3 W/(m K))	Δx_{ins}	thickness of wall insulation material (0.25 m)
k_f	thermal conductivity of floor layer material (0.2 W/(m K))	$\Delta x_{ins,c}$	thickness of ceiling insulation material (0.2 m)
k_{ins}	thermal conductivity of wall insulation material (0.032 W/(m K))	$\Delta x_{ins,s}$	thickness of spout insulation material (0.15 m)
$k_{ins,c}$	thermal conductivity of ceiling insulation material (0.032 W/(m K))	Δx_s	thickness of spout material (0.2 m)
$k_{ins,s}$	thermal conductivity of spout insulation material (0.032 W/(m K))	Δx_{wall}	thickness of wall material (0.25 m)
k_s	thermal conductivity of spout material (2.3 W/(m K))	$\varepsilon_{13}, \varepsilon_{16}$	surface emissivity of floor and ceiling (–)
k_{wall}	thermal conductivity of wall material (0.32 W/(m K))	ψ_{vent}	ventilation heat loss coefficient (W/K)
L	length of the building (10 m)	ρ_{air}	density of air (1.1945 kg/m ³)
M_i	energy accumulation (J/(m ² K))	ρ_c	density of ceiling material (2400 kg/m ³)
n_{min}	minimum air exchange rate per hour (0.5 1/h)	ρ_f	density of floor material (710 kg/m ³)
p_{air}	pressure of air (10 ⁵ Pa)	ρ_{ins}	density of wall insulation material (100 kg/m ³)
Q_{gains}	heat gains (W)	$\rho_{ins,c}$	density of ceiling insulation material (100 kg/m ³)
$Q_{gains,int}$	internal heat gains (W)	$\rho_{ins,s}$	density of spout insulation material (100 kg/m ³)
$Q_{gains,sol}$	solar gains (W)	ρ_s	density of spout material (2400 kg/m ³)
q	heat flux density (W/m ²)	ρ_{wall}	density of wall material (1200 kg/m ³)
q_{int}	thermal load of room resulting from internal heat gains (6.8 W/m ²)	σ	Stefan-Boltzmann constant (5.67 × 10 ^{–8} kg s ^{–3} K ^{–4})
R_{air}	specific gas constant (287.058 J/(kg K))		

Empirical model

a_i, b_i, c_i	parameters of the dynamic model
f^{ss}	steady-state model function
h	process disturbance (uncontrolled input)
k	discrete time (the current sampling instant)
m_0, m_1, m_2	parameters of the steady-state model
n_A, n_B, n_C	integer numbers which define order of the dynamic model
$\bar{q}$	heat flux density in the nominal operating point (5.1054 W/m ²)

$\bar{T}_{20}$	temperature T_{20} in the nominal operating point (15 °C)	$y^0(k)$	free trajectory of controlled variable
$\bar{T}_{\text{ext}}$	temperature T_{ext} in the nominal operating point (7.6 °C)	$\hat{y}(k+p k)$	predicted values of controlled variable
u	process input (manipulated variable)	$\hat{y}(k)$	predicted trajectory of controlled variable
y	process output (controlled variable)	$\Delta u(k+p k)$	future increments of manipulated variable
Model Predictive Control		$\Delta u^{\max}(k q^{\max})$	maximal change of manipulated variable
J, Λ	auxiliary matrices	$\Delta \mathbf{u}^{\max}$	vector of rate constraints of manipulated variable
N, N_u	prediction and control horizons	$\Delta \mathbf{u}(k)$	trajectory of future increments of manipulated variable
$s_i, \mathbf{G}$	step response coefficients and matrix	$\varepsilon_{\text{MPC}}^{\min}(k), \varepsilon_{\text{MPC}}^{\max}(k)$	slack variables for soft output constraints
T_{20}^{sp}	set-point of air temperature in room	$\varepsilon_{\text{MPC}}^{\min}(k), \varepsilon_{\text{MPC}}^{\max}(k)$	vectors of slack variables
T_s	sampling time	λ	weighting coefficient in MPC
$u(k+p k)$	future values of manipulated variable	$\rho_{\text{MPC}}^{\min}, \rho_{\text{MPC}}^{\max}$	weighting coefficients for slack variables
$u^{\min}(q^{\min}), u^{\max}(q^{\max})$	minimal and maximal values of manipulated variable	Optimisation	
$\mathbf{u}^{\min}, \mathbf{u}^{\max}$	vectors of magnitude constraints of manipulated variable	K_{kWh}	cost of 1 kWh of electric energy (0.13 EUR)
$y^{\min}(T_{20}^{\min}), y^{\max}(T_{20}^{\max})$	minimal and maximal values of controlled variable	K_{total}	cost of heating during the heating season (EUR)
$\mathbf{y}^{\min}, \mathbf{y}^{\max}$	vectors of magnitude constraints of controlled variable	N_{sim}	the optimisation horizon over which energy cost is calculated
$y^{\text{sp}}(k+p k)$	set-points of controlled variable	$R_{\text{GJ/kWh}}$	conversion factor from 1 GJ of heating energy to 1 kWh of electric energy (277.7778)
$\mathbf{y}^{\text{sp}}(k)$	set-point trajectory of controlled variable	ss	steady-state conditions
$y^0(k+p k)$	elements of free trajectory of controlled variable	$\varepsilon_{\text{ss}}^{\min}, \varepsilon_{\text{ss}}^{\max}$	slack variables for soft output constraints
		η_{el}	efficiency of electric heating (0.98)
		$\rho_{\text{ss}}^{\min}, \rho_{\text{ss}}^{\max}$	weighting coefficients for slack variables

experimental data.

In electric floor heating systems new heat storage materials [6] such as a solid-solid phase transition material i.e. neopentylglycol [7], paraffin-concrete and water-concrete [8] and Phase Change Materials (PCM) [9–13] are studied in order to improve the effectiveness of electrical floor heating system. Such materials could be used to store the heating energy produced in non-peak demand hours, when the electricity price is low, and use the energy in peak-demand time, when the cost is higher.

This work is concerned with modelling, Model Predictive Control (MPC) [14,15] and on-line energy optimisation of a residential building in which an underfloor heating system based on the electric heating foils is used. Important applications of MPC are concerned with building control. Typically, only temperature control (stabilisation despite changes of the outside temperature which is a disturbance) is considered, e.g. [16–18]. In more advanced solutions thermal comfort is controlled [19,20], i.e. temperature, humidity and other factors. MPC may cooperate with on-line energy optimisation which determines optimal set-points for MPC, e.g. [21–23]. In some MPC approaches weather forecasts [24] or information about occupancy behaviour [25] may be used to increase control accuracy. In some work advanced MPC techniques are used: hierarchical MPC [26], stochastic MPC [27], cooperative MPC [28], robust MPC [29].

In this work, at first, the first-principle model is developed to determine air temperature inside the building. A transient heat transfer approach employing the control volume method is used to simulate the heat transfer through the building layers. The floor heating system performance is also simulated using the variable heat flux density. The simulation allows to calculate the room-air temperature at a specified time interval. Low-order linear models for MPC are found. Tuning of MPC is discussed, both set-point change and disturbance compensation cases are discussed. Tuning is discussed for different sampling periods of MPC. Next, two control system structures are considered: MPC with a constant temperature set-point and MPC cooperating with a set-point

optimiser which repeatedly calculates on-line the set-point in order to minimise the amount of energy used. Effectiveness of these structures is validated in simulations using a real scenario of outdoor temperature changes. In general, the presented solution has two important advantages:

- Unlike previous works on underfloor heating system in which very simple on-off controllers are used, e.g. [30–33], in this work an advanced MPC algorithm which may cooperate with the set-point optimiser is used. The existing constraints imposed on the manipulated and controlled variables are taken analytically into account during on-line optimisation.
- Both MPC and set-point optimiser use on-line computationally efficient classical quadratic optimisation while in building control many MPC approaches, e.g. [34–36], and set-point optimisation methods, e.g. [21–23], use on-line computationally difficult nonlinear optimisation. Quadratic optimisation tasks have only one global solution, the problem of shallow local minima does not exists.

The major benefit of the considered system is low investment cost, significantly lower than that typical of water-based radiant floor heating. Also, there is no need to install a boiler or a heat pump. In Poland, in places with a limited access to natural gas, the heating system based on electrical energy is one option. The other one is to use air-water heat pump or water-to-water heat pump. The problem with those solutions is:

- Significant investment costs (at least 4 times higher than that of the electric radiant floor heating).
- When a water-water heat pump is used, a ground heat exchanger is needed the efficiency of which strongly depends of the type of the ground. Also additional space for a ground heat exchanger is required.

Therefore, the electric radiant heating is studied as an

alternative for the heat pump based systems.

2. Process description

The considered process is a residential building with an underfloor heating system based on the electric heating foils. Building height is $H = 2.6$ m, width is $W = 10$ m, length is $L = 10$ m. Heating surface is $A_{\text{heat}} = WL = 100 \text{ m}^2$. The underfloor electric radiant heating is applied to deliver the heating energy to the building. The 0.39 mm thick heating foil is manufactured by printing, in the honeycomb structure, carbon paste on Polyethylene Terephthalate (PET) film. The heating foil can be supplied with either alternating current (230 V) or direct current (12 V or 24 V). The maximum heat flux generated by the heating foil is equal to 200 W/m^2 .

The first-principle model is derived using the one-dimensional transient heat conduction model for determining air temperature inside building. The model considers heat transfer through walls, roof and basement. The following assumptions are made to formulate the mathematical model of the process:

1. The building is not equipped with a recuperation system. Therefore, the ventilation heat loss has a significant contribution to the entire heat loss of the building.
2. The materials of each building wall layer are considered as homogeneous with constant thermal properties. Therefore, the model results can use the values of material properties provided by the manufacturer.
3. Air density is calculated from the ideal gas law, which is quite reasonable approximation in case when the differences in air temperature and density are low.
4. Air volume in the rooms is 10% lower than the building volume calculated for internal walls.
5. Heat losses of the building including heat loss through: the envelope, the basement, windows, roof and the doors are considered. Also the ventilation heat loss is taken into account. Heat losses through thermal bridges are also considered.
6. Heat losses through the ground are modelled using an equivalent heat transfer coefficient $h_{\text{ext,g}}$ and ground temperature $T_{\text{ext,g}}$.
7. Heat convection from the air surrounding the building to the insulation material is modelled by the heat transfer coefficient h_{ext} and the external air temperature T_{ext} which is taken from the weather data for the location of Kraków city.
8. The external air temperature T_{ext} changes in time which makes it possible to carry out simulations for various temperature conditions.
9. An equivalent heat transfer coefficient h_{rad} models radiation from the floor to the ceiling internal surface of the building.
10. The walls are made of porotherm material while the ceiling and spout are made of reinforced concrete. Such a combination of wall and insulation materials is mostly valid for newly built single-family houses in Poland.
11. The insulation is made of styrofoam with a thermal conductivity k_{ins} .
12. The ceiling insulation is made of PUR foam, with a thermal conductivity $k_{\text{ins,c}}$.
13. The floor is made of oak parquet with thermal conductivity equal to k_f .
14. Heating season lasts from October 1 till April 30. This assumption is valid for Poland region.
15. A constant price of 1 kWh of electric energy is assumed and equal to 0.53 PLN = 0.13 euro [37].

16. Efficiency of the electric heating system is 98%, which is a typical value for such devices.

17. Material properties are taken from [38].

The modelling assumptions proposed allows one to calculate the heat losses during the heating period. A similar approach to modelling of building heat losses can be found in [39].

The heat transfer domain shown in Fig. 1 is subdivided into 20 nodes, which represent the building layers. Temperature in different parts of the building are denoted by $T_1, \dots, T_{20}$: T_1, T_6, T_7 and T_{12} are temperatures of the external surface of a building envelope, corresponding to north, south, east and west directions, respectively, T_2, T_5, T_8 and T_{11} are temperatures of the interface between insulation layer and building wall, corresponding to north, south, east and west directions, respectively, T_3, T_4, T_9 and T_{10} are temperatures of the inner wall surface of the building, corresponding to north, south, east and west directions, respectively, T_{13} is temperature of the inner surface of the building roof, T_{14} is temperature of the inner surface of the building roof insulation, T_{15} is temperature of the outer surface of the building roof insulation, T_{16} is temperature of the inner surface of the flooring, T_{17} is temperature of the outer surface of the flooring, T_{18} is temperature of the outer surface of the spout, T_{19} is temperature of the outer surface of the basement insulation, T_{20} is air temperature in the room.

The control volume method is used to formulate the heat balance equations for each node. The one dimensional transient heat conduction equation is solved to obtain nodal temperatures $T_1, \dots, T_{20}$. The electrical heating foil is applied below the spout layer. Therefore, the heat source generated by the electric heating component equal at the maximum of 200 W/m^2 is added in the equation of node 18. The set of ordinary differential equations which comprise the model is solved by the Runge-Kutta method of order 45.

From the point of view of control engineering, the considered process has one manipulated (input) variable q , one disturbance (uncontrolled input) variable T_{ext} and one controlled (output) variable T_{20} . The control objective is to calculate repeatedly on-line the value of the manipulated variable, q , in such a way that the temperature inside the building, T_{20} , is close to its set-point value, T_{20}^{sp} , despite the fact that the external temperature changes in time. Additionally, the controller must be able to quickly react when the user changes the set-point. The developed mathematical model allows one to simulate the thermal performance of the applied radiant heating, as well as to perform the economic analysis of the underground electric heating system operation.

The first-principle mathematical model is based on the Finite Volume Method (FVM). The FVM method is used to discretize the transient heat conduction equation. The model considers 20 nodes for which the energy balance equations are formulated. Nodes 1, 6, 7 and 12 describe external surface of the building envelope

$$M_1 \frac{dT_1}{dt} = h_{\text{ext}}(T_{\text{ext}} - T_1) + \frac{k_{\text{ins}}}{\Delta x_{\text{ins}}}(T_2 - T_1) \quad (1)$$

$$M_6 \frac{dT_6}{dt} = h_{\text{ext}}(T_{\text{ext}} - T_6) + \frac{k_{\text{ins}}}{\Delta x_{\text{ins}}}(T_5 - T_6) \quad (2)$$

$$M_7 \frac{dT_7}{dt} = h_{\text{ext}}(T_{\text{ext}} - T_7) + \frac{k_{\text{ins}}}{\Delta x_{\text{ins}}}(T_8 - T_7) \quad (3)$$

$$M_{12} \frac{dT_{12}}{dt} = h_{\text{ext}}(T_{\text{ext}} - T_{12}) + \frac{k_{\text{ins}}}{\Delta x_{\text{ins}}}(T_{11} - T_{12}) \quad (4)$$

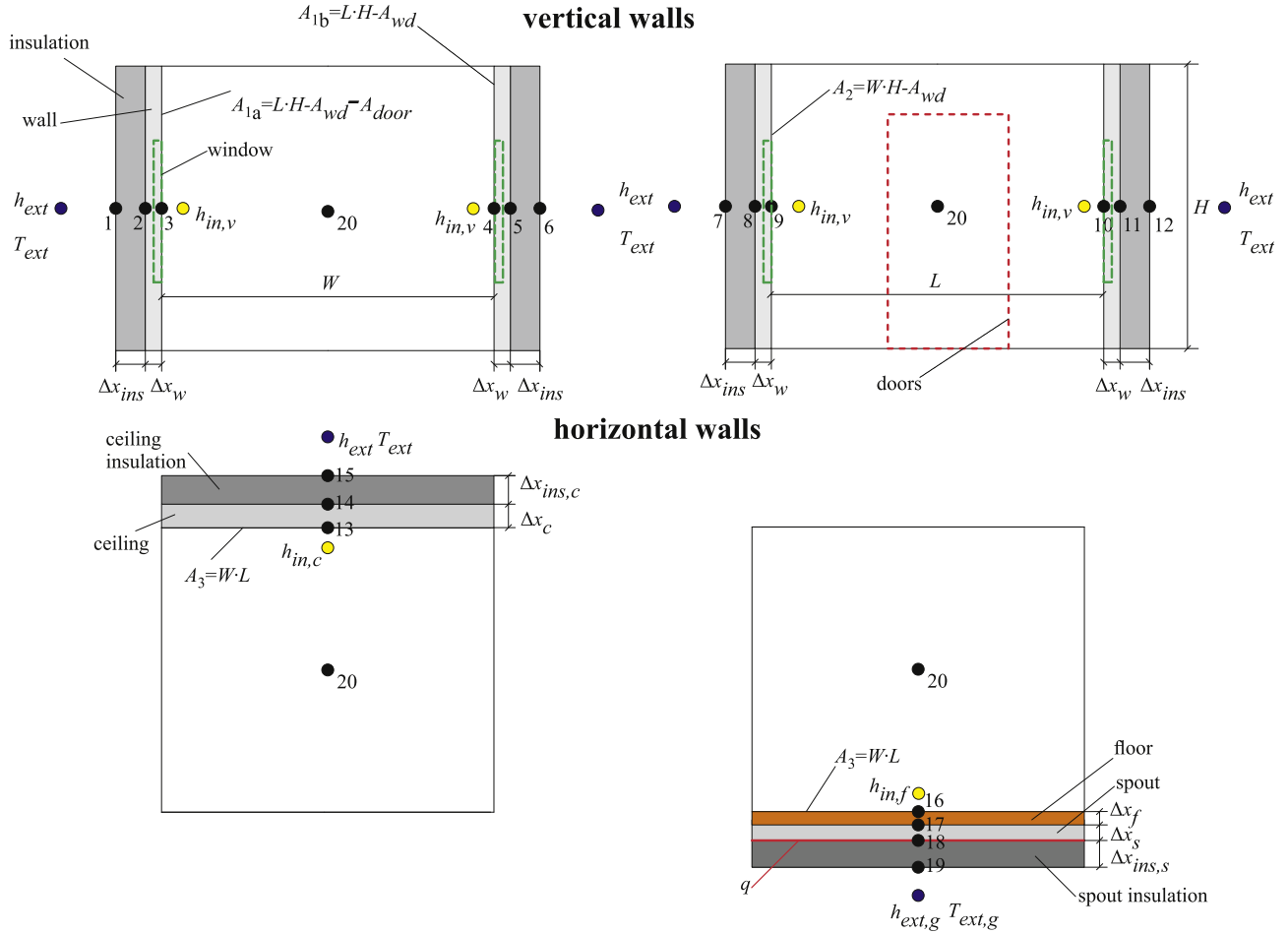


Fig. 1. A residential building with an underfloor heating system based on the electric heating foils.

where energy accumulation is

$$M_i = \rho_{ins} c_{ins} \frac{\Delta x_{ins}}{2} \quad (5)$$

for $i = 1, 6, 7, 12$, ρ_{ins} is density of wall insulation material, c_{ins} is specific heat of wall insulation material, Δx_{ins} is thickness of wall insulation material, h_{ext} is heat transfer coefficient from the external air, k_{ins} is thermal conductivity of wall insulation material.

Nodes 2, 5, 8 and 11 describe interface between the internal surface of insulation layer and building wall external surface

$$M_2 \frac{dT_2}{dt} = \frac{k_{ins}}{\Delta x_{ins}} (T_1 - T_2) + \frac{k_{wall}}{\Delta x_{wall}} (T_3 - T_2) \quad (6)$$

$$M_5 \frac{dT_5}{dt} = \frac{k_{ins}}{\Delta x_{ins}} (T_6 - T_5) + \frac{k_{wall}}{\Delta x_{wall}} (T_4 - T_5) \quad (7)$$

$$M_8 \frac{dT_8}{dt} = \frac{k_{ins}}{\Delta x_{ins}} (T_7 - T_8) + \frac{k_{wall}}{\Delta x_{wall}} (T_9 - T_8) \quad (8)$$

$$M_{11} \frac{dT_{11}}{dt} = \frac{k_{ins}}{\Delta x_{ins}} (T_{12} - T_{11}) + \frac{k_{wall}}{\Delta x_{wall}} (T_{10} - T_{11}) \quad (9)$$

where energy accumulation is

$$M_i = \rho_{ins} c_{ins} \frac{\Delta x_{ins}}{2} + \rho_{wall} c_{wall} \frac{\Delta x_{wall}}{2} \quad (10)$$

for $i = 2, 5, 8, 11$, ρ_{wall} is density of wall material, c_{wall} is specific heat of wall material, Δx_{wall} is thickness of wall material, k_{wall} is thermal conductivity of wall material.

Nodes 3, 4, 9 and 10 describe internal surface of the walls of the building

$$M_3 \frac{dT_3}{dt} = \frac{k_{wall}}{\Delta x_{wall}} (T_2 - T_3) + h_{in,v} (T_{20} - T_3) \quad (11)$$

$$M_4 \frac{dT_4}{dt} = \frac{k_{wall}}{\Delta x_{wall}} (T_5 - T_4) + h_{in,v} (T_{20} - T_4) \quad (12)$$

$$M_9 \frac{dT_9}{dt} = \frac{k_{wall}}{\Delta x_{wall}} (T_8 - T_9) + h_{in,v} (T_{20} - T_9) \quad (13)$$

$$M_{10} \frac{dT_{10}}{dt} = \frac{k_{wall}}{\Delta x_{wall}} (T_{11} - T_{10}) + h_{in,v} (T_{20} - T_{10}) \quad (14)$$

where energy accumulation is

$$M_i = \rho_{wall} c_{wall} \frac{\Delta x_{wall}}{2} \quad (15)$$

for $i = 3, 4, 9, 10$, $h_{in,v}$ is an indoor air heat transfer coefficient determined for vertical walls.

Node 13 describes ceiling surface

$$M_{13} \frac{dT_{13}}{dt} = \frac{k_c}{\Delta x_c} (T_{14} - T_{13}) + h_{in,c} (T_{20} - T_{13}) + h_{rad} (T_{16} - T_{13}) \quad (16)$$

where energy accumulation is

$$M_{13} = \rho_c c_c \frac{\Delta x_c}{2} \quad (17)$$

ρ_c is density of ceiling material, c_c is specific heat of ceiling material, Δx_c is thickness of ceiling material, k_c is thermal conductivity of ceiling material, $h_{in,c}$ is an indoor air heat transfer coefficient for ceiling surface. Radiation heat transfer coefficient is defined as

$$h_{rad} = \frac{\sigma (T_{16}^2 + T_{13}^2) (T_{16} + T_{13})}{\frac{1}{\varepsilon_{13}} + \frac{1}{\varepsilon_{16}} - 1} \quad (18)$$

where σ is Stefan-Boltzmann constant, ε_{13} and ε_{16} are the surface emissivity factors of floor and ceiling, respectively.

Node 14 describes interface between the internal surface of ceiling insulation layer and ceiling external surface

$$M_{14} \frac{dT_{14}}{dt} = \frac{k_c}{\Delta x_c} (T_{13} - T_{14}) + \frac{k_{ins,c}}{\Delta x_{ins,c}} (T_{15} - T_{14}) \quad (19)$$

where energy accumulation is

$$M_{14} = \rho_{ins,c} c_{ins,c} \frac{\Delta x_{ins,c}}{2} \quad (20)$$

where $\rho_{ins,c}$ is density of ceiling insulation material, $c_{ins,c}$ is specific heat of ceiling insulation material, $k_{ins,c}$ is thermal conductivity of ceiling insulation material, $\Delta x_{ins,c}$ is thickness of ceiling insulation material.

Node 15 describes the external surface of the ceiling insulation

$$M_{15} \frac{dT_{15}}{dt} = \frac{k_{ins,c}}{\Delta x_{ins,c}} (T_{14} - T_{15}) + h_{ext} (T_{ext} - T_{15}) \quad (21)$$

where energy accumulation is

$$M_{15} = \rho_{ins,c} c_{ins,c} \frac{\Delta x_{ins,c}}{2} \quad (22)$$

Node 16 describes the surface of the floor panels

$$M_{16} \frac{dT_{16}}{dt} = \frac{k_f}{\Delta x_f} (T_{17} - T_{16}) + h_{in,f} (T_{20} - T_{16}) + h_{rad} (T_{13} - T_{16}) \quad (23)$$

where energy accumulation is

$$M_{16} = \rho_f c_f \frac{\Delta x_f}{2} \quad (24)$$

ρ_f is density of floor layer, c_f is specific heat of floor layer, k_f is thermal conductivity of floor layer, Δx_f is thickness of floor layer, $h_{in,f}$ is an indoor air heat transfer coefficient for floor surface.

Node 17 describes interface between floor and spout layer

$$M_{17} \frac{dT_{17}}{dt} = \frac{k_f}{\Delta x_f} (T_{16} - T_{17}) + \frac{k_s}{\Delta x_s} (T_{18} - T_{17}) \quad (25)$$

where energy accumulation is

$$M_{17} = \rho_f c_f \frac{\Delta x_f}{2} + \rho_s c_s \frac{\Delta x_s}{2} \quad (26)$$

ρ_s is density of spout material, c_s is specific heat of spout material, Δx_s is thickness of spout material, k_s is thermal conductivity of spout material.

Node 18 describes interface between spout layer and spout insulation

$$M_{18} \frac{dT_{18}}{dt} = \frac{k_s}{\Delta x_s} (T_{17} - T_{18}) + \frac{k_{ins,s}}{\Delta x_{ins,s}} (T_{19} - T_{18}) + q \quad (27)$$

where energy accumulation is

$$M_{18} = \rho_{ins,s} c_{ins,s} \frac{\Delta x_{ins,s}}{2} + \rho_s c_s \frac{\Delta x_s}{2} \quad (28)$$

where $\rho_{ins,s}$ is density of spout insulation material, $c_{ins,s}$ is specific heat of spout insulation material, $\Delta x_{ins,s}$ is thickness of spout insulation material, $k_{ins,s}$ is thermal conductivity of spout insulation material.

Node 19 describes the external surface of the basement insulation

$$M_{19} \frac{dT_{19}}{dt} = \frac{k_{ins,s}}{\Delta x_{ins,s}} (T_{18} - T_{19}) + h_{ext,g} (T_{ext,g} - T_{19}) \quad (29)$$

where energy accumulation is

$$M_{19} = \rho_{ins,s} c_{ins,s} \frac{\Delta x_{ins,s}}{2} \quad (30)$$

$h_{ext,g}$ is ground overall heat transfer coefficient, $T_{ext,g}$ is ground temperature.

Node 20 describes internal air

$$\begin{aligned} M_{20} \frac{dT_{20}}{dt} = & h_{in,v} A_{1a} (T_3 - T_{20}) + h_{in,v} A_{1b} (T_4 - T_{20}) \\ & + h_{in,v} A_2 (T_9 - T_{20}) + h_{in,v} A_2 (T_{10} - T_{20}) \\ & + h_{in,f} A_3 (T_{16} - T_{20}) + h_{in,c} A_3 (T_{13} - T_{20}) \\ & + 4U_{wd} A_{wd} (T_{ext} - T_{20}) + U_{door} A_{door} (T_{ext} - T_{20}) \\ & + \psi_{vent} (T_{ext} - T_{20}) + Q_{gains} \end{aligned} \quad (31)$$

where energy accumulation is

$$M_{20} = \rho_{air} c_{air} V_{air} \quad (32)$$

$\rho_{air} = p_{air} / (R_{air} (T_{20} + 273.15))$ is density of air (p_{air} is pressure of air, R_{air} is specific gas constant), c_{air} is air specific heat capacity. Volume of air in the room is

$$V_{air} = 0.9LHW \quad (33)$$

Vertical areas of walls (Fig. 1) are calculated from

$$A_{1a} = LH - A_{wd} - A_{door} \quad (34)$$

$$A_{1b} = LH - A_{wd} \quad (35)$$

$$A_2 = WH - A_{wd} \quad (36)$$

where A_{wd} is total area of windows at the building wall, A_{door} is area of entrance door. Horizontal wall area is calculated as

$$A_3 = WL \quad (37)$$

U_{door} and U_{wd} are overall heat transfer coefficients of doors and windows, respectively. Those values consider the existence of thermal bridges. ψ_{vent} is a ventilation heat loss coefficient calculated as

$$\psi_{vent} = 0.34V_i \quad (38)$$

Volumetric flow rate of air is

$$V_i = n_{min} V_{air} \quad (39)$$

where n_{min} is a minimum air exchange rate per hour.

In Eq. (31) heat gains are considered. They are calculated as [40]

$$Q_{gains} = Q_{gains,sol} + Q_{gains,int} \quad (40)$$

Internal heat gains are

$$Q_{gains,int} = q_{int} A_3 \quad (41)$$

where q_{int} is a thermal load of room resulting from internal heat gains assumed for a single family house. In the heating period, for single-family houses, the average contribution of solar gains is approx. 50% smaller than that of the internal gains. Therefore, it is assumed that

$$Q_{gains} = 1.5Q_{gains,int} \quad (42)$$

3. Model Predictive Control problem formulation

The universal notation is: the input of the dynamic process (the manipulated variable of the control algorithm) is u , the output (the controlled variable) is y and the disturbance (the uncontrolled input) is h .

MPC is a computer control algorithm [14,15]. Calculations are carried out in discrete time, i.e. they are repeated on-line, the time between two consecutive repetitions of the controller is named the sampling period and denoted by T_s . In the MPC algorithms not only the value of the manipulated variable for the current sampling instant, i.e. $u(k)$, but a future sequence of that variable over the control horizon N_u is repeatedly determined on-line. Typically, the control policy is defined by the vector of increments

$$\Delta u(k) = [\Delta u(k|k) \dots \Delta u(k+N_u-1|k)]^T \quad (43)$$

where $\Delta u(k|k) = u(k|k) - u(k-1)$ and $\Delta u(k+p|k) = u(k+p|k) - u(k+p-1|k)$ for $p \geq 1$, respectively. The symbol $u(k+p|k)$ denotes the value of the manipulated variable for the future sampling instant $k+p$ calculated at the current instant k . It is assumed that $\Delta u(k+p|k) = 0$ for $p \geq N_u$. The future sequence of increments of the manipulated variable (43) is calculated from an optimisation problem in which the future control quality is minimized. The typical MPC cost-function is

$$J(k) = \sum_{p=1}^N \|y^{sp}(k+p|k) - \hat{y}(k+p|k)\|^2 + \lambda \sum_{p=0}^{N_u-1} \|\Delta u(k+p|k)\|^2 \quad (44)$$

The set-point for the sampling instant $k+p$ known at the current instant k is $y^{sp}(k+p|k)$ (usually $y^{sp}(k+p|k) = y^{sp}(k)$ for all $p = 1, \dots, N$), the future value of the process output signal predicted for the instant $k+p$ at the instant k is denoted by $\hat{y}(k+p|k)$. Hence, the first part of the minimised cost-function measures the control error calculated on the prediction horizon N . The second part of the cost-function penalises excessive increments of the manipulated variable. The bigger the weighting coefficient $\lambda > 0$, the smaller the increments, but also the slower the changes of the process output.

In practice the MPC cost function (44) is minimised subject to some constraints imposed on process variables. The rudimentary constrained MPC optimisation task solved on-line at each sampling instant is

$$\begin{aligned} & \min_{\Delta u(k)} \{J(k)\} \\ & \text{subject to} \\ & u^{\min} \leq u(k+p|k) \leq u^{\max}, \quad p = 0, \dots, N_u - 1 \\ & -\Delta u^{\max} \leq \Delta u(k+p|k) \leq \Delta u^{\max}, \quad p = 0, \dots, N_u - 1 \\ & y^{\min} \leq \hat{y}(k+p|k) \leq y^{\max}, \quad p = 1, \dots, N \end{aligned} \quad (45)$$

In the above formulation it is possible to limit the values of the manipulated variable over the control horizon, $u^{\min}$ and $u^{\max}$ are the minimal and the maximal allowed values, respectively. Moreover, the increments of the manipulated variable, also over the control horizon, are limited, $\Delta u^{\max}$ is the maximal allowed change possible in one sampling instant. Finally, it may be necessary to limit the predicted values of the controlled variable over the prediction horizon, $y^{\min}$ and $y^{\max}$ denote the minimal and the maximal allowed values.

The constraints imposed on the value of the manipulated variable limit the region of possible solutions of the MPC optimisation problem (45), but the constraints imposed on the value of the predicted process output may lead to an acceptable situation in which the set of possible solutions is empty. In order to enlarge the admissible set the output constraints are relaxed when necessary. The resulting MPC optimisation task with soft output constraints is

$$\begin{aligned} & \min_{\substack{\Delta u(k) \\ \epsilon_{MPC}^{\min}(k) \\ \epsilon_{MPC}^{\max}(k)}} \left\{ J(k) + \rho_{MPC}^{\min} (\epsilon_{MPC}^{\min}(k))^2 + \rho_{MPC}^{\max} (\epsilon_{MPC}^{\max}(k))^2 \right\} \\ & \text{subject to} \\ & u^{\min} \leq u(k+p|k) \leq u^{\max}, \quad p = 0, \dots, N_u - 1 \\ & -\Delta u^{\max} \leq \Delta u(k+p|k) \leq \Delta u^{\max}, \quad p = 0, \dots, N_u - 1 \\ & y^{\min} - \epsilon_{MPC}^{\min}(k) \leq \hat{y}(k+p|k) \leq y^{\max} + \epsilon_{MPC}^{\max}(k), \quad p = 1, \dots, N \\ & \epsilon_{MPC}^{\min}(k) \geq 0, \quad \epsilon_{MPC}^{\max}(k) \geq 0 \end{aligned} \quad (46)$$

where $\epsilon_{MPC}^{\min}(k)$ and $\epsilon_{MPC}^{\max}(k)$ are additional slack variables which are set as greater than 0 when the output hard constraints must be violated. The degree of constraints' violation is minimised by the

last two penalty terms in the minimised MPC cost-function, $\rho_{\text{MPC}}^{\min}$ and $\rho_{\text{MPC}}^{\max}$ are positive weighting coefficients.

Solution of the MPC optimisation task (46) yields the future control increments (43), but only the first element of this sequence is actually applied to the process, i.e. $u(k) = \Delta u(k|k) + u(k-1)$. At

called free trajectory, i.e. the reaction of the model (47) to the past. Using the prediction equation (49), the MPC optimisation problem with soft output constraints (46) is transformed to the following quadratic optimisation task

$$\min_{\substack{\Delta \mathbf{u}(k) \\ \epsilon_{\text{MPC}}^{\min}(k) \\ \epsilon_{\text{MPC}}^{\max}(k)}} \left\{ \left\| \mathbf{y}^{\text{sp}}(k) - \mathbf{G} \Delta \mathbf{u}(k) - \mathbf{y}^0(k) \right\|^2 + \left\| \Delta \mathbf{u}(k) \right\|_{\Lambda}^2 + \rho_{\text{MPC}}^{\min} \left(\epsilon_{\text{MPC}}^{\min}(k) \right)^2 + \rho_{\text{MPC}}^{\max} \left(\epsilon_{\text{MPC}}^{\max}(k) \right)^2 \right\} \quad (51)$$

subject to

$$\begin{aligned} \mathbf{u}^{\min} &\leq \mathbf{J} \Delta \mathbf{u}(k) + \mathbf{u}(k-1) \leq \mathbf{u}^{\max} \\ -\Delta \mathbf{u}^{\max} &\leq \Delta \mathbf{u}(k) \leq \Delta \mathbf{u}^{\max} \\ \mathbf{y}^{\min} - \epsilon_{\text{MPC}}^{\min}(k) &\leq \mathbf{G} \Delta \mathbf{u}(k) + \mathbf{y}^0(k) \leq \mathbf{y}^{\max} + \epsilon_{\text{MPC}}^{\max}(k) \end{aligned}$$

the next sampling instant, $k+1$, the measurement of the process output is updated, the prediction is shifted one step forward and the whole procedure is repeated.

MPC algorithms have been used for years in industrial process control, e.g. to control chemical reactors and distillation columns [41]. Currently, in addition to building control implementations mentioned in introduction, applications of MPC algorithms also include energy-related processes, e.g.: a fuel cell [42], a nuclear superheated-steam supply system [43], a refrigeration system [44], a boiler-turbine system [45], a heat exchanger [46], a gas turbine [47], a wind turbine [48], a heat pump [49] and microgrids [50].

In this work the Generalized Predictive Control (GPC) algorithm with the so called Dynamic Matrix Control (DMC) disturbance model is used [15]. For prediction, i.e. to calculate the values of $\hat{y}(k+p|k)$ for $p = 1, \dots, N$, the following discrete-time linear dynamic model is used

$$\begin{aligned} y(k) &= b_1 u(k-1) + \dots + b_{n_b} u(k-n_b) \\ &+ c_1 h(k-1) + \dots + c_{n_c} h(k-n_c) \\ &- a_1 y(k-1) - \dots - a_{n_a} y(k-n_a) \end{aligned} \quad (47)$$

The scaled process input, disturbance and output signals are

$$u = q - \bar{q}, \quad d = T_{\text{ext}} - \bar{T}_{\text{ext}}, \quad y = T_{20} - \bar{T}_{20} \quad (48)$$

where the values of process variables in the nominal operating point are: $\bar{T}_{20}$, $\bar{q}$, $\bar{T}_{\text{ext}}$. The order of the dynamic model is determined by the integer numbers n_a , n_b , n_c , the real numbers a_i , b_i , c_i are model parameters. In general, a great advantage of linear models is the fact that the prediction vector, defined over the prediction horizon, N , is a linear function of the MPC decision variables, $\Delta \mathbf{u}(k)$. In case of the GPC algorithm it is possible to prove that [15]

$$\hat{\mathbf{y}}(k) = [\hat{y}(k+1|k) \dots \hat{y}(k+N|k)]^T = \mathbf{G} \Delta \mathbf{u}(k) + \mathbf{y}^0(k) \quad (49)$$

where the dynamic matrix of dimensionality $N \times N_u$ consists of step-response coefficients s_i of the process model (47)

$$\mathbf{G} = \begin{bmatrix} s_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_N & s_{N-1} & \dots & s_{N-N_u+1} \end{bmatrix} \quad (50)$$

The vector $\mathbf{y}^0(k) = [y^0(k+1|k) \dots y^0(k+N|k)]^T$ represents the so

where the vectors of length N are: $\mathbf{y}^{\text{sp}} = [y^{\text{sp}}(k+1|k) \dots y^{\text{sp}}(k+N|k)]^T$, $\mathbf{y}^{\min} = [y^{\min} \dots y^{\min}]^T$, $\mathbf{y}^{\max} = [y^{\max} \dots y^{\max}]^T$, $\epsilon_{\text{MPC}}^{\min}(k) = [\epsilon_{\text{MPC}}^{\min}(k) \dots \epsilon_{\text{MPC}}^{\min}(k)]^T$ and $\epsilon_{\text{MPC}}^{\max}(k) = [\epsilon_{\text{MPC}}^{\max}(k) \dots \epsilon_{\text{MPC}}^{\max}(k)]^T$. The vectors of length N_u are: $\mathbf{u}^{\min} = [u^{\min} \dots u^{\min}]^T$, $\mathbf{u}^{\max} = [u^{\max} \dots u^{\max}]^T$, $\Delta \mathbf{u}^{\max} = [\Delta u^{\max} \dots \Delta u^{\max}]^T$ and $\mathbf{u}(k-1) = [u(k-1) \dots u(k-1)]^T$. The diagonal matrix $\Lambda = \text{diag}(\lambda, \dots, \lambda)$ and the lower triangular matrix $\mathbf{J}$ are of dimensionality $N_u \times N_u$.

It is important to stress the fact that the obtained MPC problem solved at each sampling instant on-line (51) is of a quadratic optimisation type. It is possible because the model used for prediction (Eq. (47)) is linear and the predicted trajectory (Eq. (49)) is a linear function of the optimised control sequence. The classical active-set method [51] is used to solve the MPC problem. However, the basic question if whether or not the nonlinear fundamental model described in Section 2 may be approximated by the linear model (47).

4. Energy optimisation

The objective of energy optimisation is to find the value of the temperature set-point, T_{20}^{sp} , for which cost of energy used is minimal. It cannot be done once, off-line, but must be recalculated successively on-line, preferably at each sampling instant of the MPC algorithm. The necessity of set-point recalculation is caused by the fact that the process disturbance, the external temperature, T_{ext} , is not constant during the whole heating season.

In addition to the necessity of minimisation of energy cost, it is also required to control the process in such a way that the air temperature in the building, T_{20} , is in some neighbourhood of the preferable temperature T_{20}^{sp} . Two preferable temperatures are considered: $T_{20}^{\text{sp}} = 18.5^\circ \text{C}$ and $T_{20}^{\text{sp}} = 21^\circ \text{C}$. In the first case the constraints imposed on the predicted process output are determined by

$$T_{20}^{\min} = 18^\circ \text{C}, \quad T_{20}^{\max} = 19^\circ \text{C} \quad (52)$$

whereas in the second case the constraints are

$$T_{20}^{\min} = 20.5^\circ \text{C}, \quad T_{20}^{\max} = 21.5^\circ \text{C} \quad (53)$$

In general, the unit cost of heating during the whole heating season is calculated as

$$K_{\text{total}} = \frac{K_{\text{kWh}} R_{\text{GJ/kWh}} A_{\text{heat}} T_s}{10^9 \eta_{\text{el}}} \sum_{p=0}^{N_{\text{sim}}-1} q(k+p) \quad (54)$$

where K_{kWh} is the cost of 1 kWh of electric energy, $R_{\text{GJ/kWh}}$ is the conversion factor from 1 GJ of heating energy to 1 kWh of electric energy, $A_{\text{heat}} = WL$ is total heated area, N_{sim} is the optimisation horizon over which energy cost is calculated, η_{el} is efficiency of electric heating.

Typically, for set-point optimisation a steady-state model of the process is used (Steady-State Target Optimisation, SSTO) [52,53]. For the considered problem, it has the following form

$$T_{20}^{\text{ss}} = f^{\text{ss}}(q^{\text{ss}}, T_{\text{ext}}^{\text{ss}}) \quad (55)$$

where T_{20}^{ss} , q^{ss} and $T_{\text{ext}}^{\text{ss}}$ denote values of the process output, input and disturbance signals in the steady-state, respectively, f^{ss} denotes the steady-state model (the superscript “ss” is used to denote the steady-state of process variables).

Taking into account that the objective is to minimise energy cost, assuming that the cost of energy is constant in time and the constraints imposed on room temperature (Eq. (52) or (53)) must be satisfied, the optimisation problem solved by the SSTO layer is

$$\begin{aligned} \min_{q^{\text{ss}}} \{J_E = q^{\text{ss}}\} \\ \text{subject to} \\ q^{\min} \leq q^{\text{ss}} \leq q^{\max} \\ T_{20}^{\min} \leq T_{20}^{\text{ss}} \leq T_{20}^{\max} \\ T_{20}^{\text{ss}} = f^{\text{ss}}(q^{\text{ss}}, T_{\text{ext}}^{\text{ss}}) \end{aligned} \quad (56)$$

Let us discuss in details the SSTO problem (56). The decision variable q^{ss} is the value of the manipulated variable (heat flux density) in the steady-state. The minimised cost-function J_E is the value of the process manipulated variable in steady-state. It has a straightforward interpretation: the smaller the manipulated variable, the lower the energy cost (Eq. (54)). Optimisation is performed subject to three constraints. The first one limits the value of the manipulated variable. The second constraint limits the value of the process output, i.e. the temperature T_{20} . Finally, the steady-state model (53) is used as the last constraint.

The rudimentary SSTO optimisation problem (56) may be reformulated for the scaled process variables u, h, y (Eq. (48)). It has the following form

$$\begin{aligned} \min_{u^{\text{ss}}} \{J_E = u^{\text{ss}}\} \\ \text{subject to} \\ u^{\min} \leq u^{\text{ss}} \leq u^{\max} \\ y^{\min} \leq y^{\text{ss}} \leq y^{\max} \\ y^{\text{ss}} = f^{\text{ss}}(u^{\text{ss}}, h^{\text{ss}}) \end{aligned} \quad (57)$$

The type of the obtained set-point optimisation problem solved at each sampling instant on-line depends on the type of the steady-state model (Eq. (55)). In order to obtain a computationally efficient solution, the following linear steady-state model is assumed

$$y^{\text{ss}} = m_0 + m_1 u^{\text{ss}} + m_2 h^{\text{ss}} \quad (58)$$

where m_0, m_1, m_2 are coefficients. Taking into account the model (58) and using the soft output constraints, from the general set-point optimisation task (57) the following quadratic optimisation

problem is obtained

$$\begin{aligned} \min_{\substack{u^{\text{ss}} \\ \epsilon_{\text{ss}}^{\min} \\ \epsilon_{\text{ss}}^{\max}}} \{J_E = u^{\text{ss}} + \rho_{\text{ss}}^{\min} (\epsilon_{\text{ss}}^{\min})^2 + \rho_{\text{ss}}^{\max} (\epsilon_{\text{ss}}^{\max})^2\} \\ \text{subject to} \\ u^{\min} \leq u^{\text{ss}} \leq u^{\max} \\ y^{\min} - \epsilon_{\text{ss}}^{\min} \leq y^{\text{ss}} \leq y^{\max} + \epsilon_{\text{ss}}^{\max} \\ y^{\text{ss}} = m_0 + m_1 u^{\text{ss}} + m_2 h^{\text{ss}} \\ \epsilon_{\text{ss}}^{\min} \geq 0, \epsilon_{\text{ss}}^{\max} \geq 0 \end{aligned} \quad (59)$$

The set-point optimisation problem (59) optimises not only the decision variable u^{ss} , but it also calculates necessary degree of violation of the original output constraints, specified by the quantities $\epsilon_{\text{ss}}^{\min}$ and $\epsilon_{\text{ss}}^{\max}$. Additionally, two penalty terms in the MPC cost-function have the objective of minimising the degree out constraints' violation ($\rho_{\text{ss}}^{\min}$ and $\rho_{\text{ss}}^{\max}$ are positive weighting coefficients). Similarly to the MPC task (51), the resulting set-point task may be also solved by means of the classical active-set method [51]. However, the basic question is whether or not the linear steady-state model (58) approximates behaviour of the nonlinear process detailed in Section 2.

It is necessary to stress the fact that the limits imposed on both manipulated and controlled variables, respectively, are the same in the set-point optimisation problem (59) and in the MPC optimisation task (51).

As a result of set-point optimisation performed at each sampling instant the value of u^{ss} is obtained. Next, the steady-state model is used to find the optimal set-point $y^{\text{sp}}(k)$

$$y^{\text{sp}}(k) = m_0 + m_1 u^{\text{ss}} + m_2 h^{\text{ss}} \quad (60)$$

which is next used in the MPC optimisation problem (51).

5. Simulation results

5.1. Process modelling for MPC

At first, models which are used in the GPC algorithm for prediction calculation must be found. Considering the discrete-time model (47), it is necessary to determine model order and model parameters. The model used in MPC is empirical, i.e. its parameters are obtained from an identification procedure. For this purpose, the fundamental continuous-time state-space model described in Section 2 is excited by some sequences of the manipulated variable, q , and the disturbance variable, T_{ext} . The resulting sequence of the temperature T_{20} is recorded. Two sets of data are prepared, i.e. the training and validation data sets which are shown in Fig. 2. The first set is used only for model identification, i.e. to find parameters of empirical models, the second one to compare different models and choose the best one. Model parameters are calculated from an unconstrained optimisation problem in which the following model error is minimised

$$E = \frac{1}{n_{\text{samples}}} \sum_{k=1}^{n_{\text{samples}}} (y_{\text{mod}}(k) - y(k))^2 \quad (61)$$

The cost-function (61) is the sum of squared differences between the model output, $y_{\text{mod}}(k)$, and the recorded value of the process output, $y(k)$, for all available samples, i.e. for $k = 1, \dots, n_{\text{samples}}$. The sum of squared model errors are divided by the number of samples. Because of that it is possible to compare models in which different sampling periods (and different numbers of data samples) are used for model identification and validation. The first-

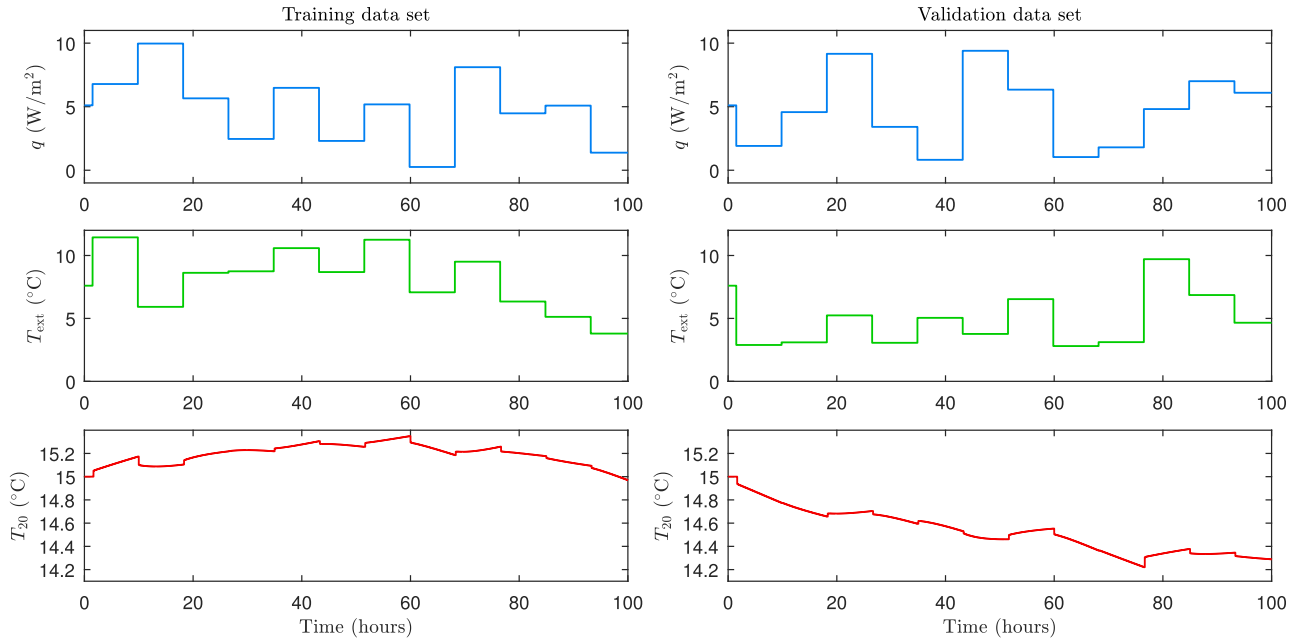


Fig. 2. The training and validation data sets.

principle model described in Section 2 is treated as the real process. Since the empirical model (47) is linear and the cost-function (61) is quadratic, optimisation task of model parameters becomes an easy to solve least-squares problem.

Table 1 compares errors of empirical models for different sampling periods, $T_s = 60, 300, 600$ s, and for different order of dynamics (the order is defined as the integral number $n_A = n_B = n_C$). For the shortest sampling period, $T_s = 60$ s, taking into account only the validation error, the best model is characterised by the fifth order of dynamics. Fig. 3 depicts the validation data vs. the output of the first-order, second-order, third-order, fourth-order and fifth-order models. Indeed, even in the case of the fourth-order model there is some difference between the data and the model output, it is really necessary to use the fifth-order model. The output of the chosen fifth-order model is very close to the data. For the sampling period $T_s = 300$ s the lowest value of the validation error is obtained for the fourth-order model. Fig. 4 shows the validation data vs. the output of the first-order, second-order, third-order and

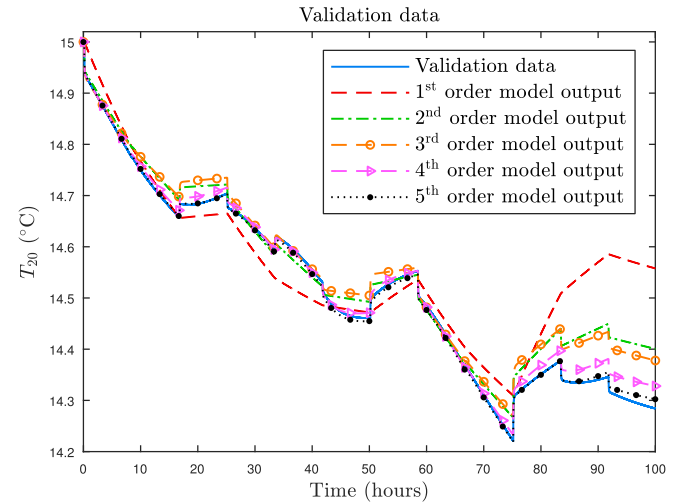


Fig. 3. The validation data vs. the output of the first-order, second-order, third-order, fourth-order and fifth-order models; $T_s = 60$ s.

Table 1
Errors of models for different sampling periods, T_s , and for different order of dynamics: the chosen model in each class is emphasised.

Model order	T_s (s)	E_{train}	E_{val}
2	60	1.6327×10^{-3}	2.2898×10^{-3}
3	60	1.5309×10^{-3}	2.0133×10^{-3}
4	60	2.4980×10^{-4}	2.6375×10^{-4}
5	60	2.0908×10^{-5}	3.9492×10^{-5}
6	60	5.1472×10^{-6}	4.2161×10^{-5}
2	300	1.5803×10^{-3}	2.1196×10^{-3}
3	300	8.5121×10^{-4}	1.3304×10^{-3}
4	300	1.0205×10^{-5}	2.7426×10^{-5}
5	300	3.5276×10^{-6}	4.1862×10^{-5}
6	300	2.9591×10^{-6}	4.5757×10^{-5}
2	600	1.5260×10^{-3}	2.0408×10^{-3}
3	600	3.7043×10^{-6}	9.3153×10^{-6}
4	600	2.4440×10^{-6}	3.5955×10^{-5}
5	600	2.4540×10^{-6}	4.4330×10^{-5}
6	600	2.3613×10^{-6}	4.5836×10^{-5}

fourth-order models. In this case the fourth order of dynamics is really necessary since even the third-order model gives noticeable modelling differences between the data and the model output. For the longest sampling period, $T_s = 600$ s, the lowest value of the validation error is obtained for the third-order model. Fig. 5 compares the data and the output of the first-order, second-order and third-order models. Also in this case, the models characterised by the order of dynamics lower than the selected ones give significant modelling errors. All things considered, although the rudimentary fundamental dynamic model presented in Section 2 is nonlinear, linear models of relatively low order are able to approximate behaviour of the process very well, which means that it is natural to use the MPC algorithm which solves at each sampling instant the quadratic optimisation problem (51), nonlinear optimisation is not necessary. The order of dynamics of the empirical model which is next used for prediction in MPC must be selected taking into

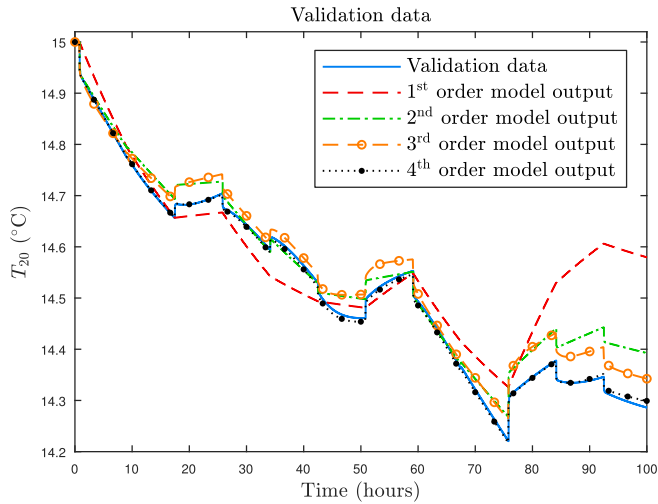


Fig. 4. The validation data vs. the output of the first-order, second-order, third-order and fourth-order models; $T_s = 300$ s.

account the sampling period. In general, the shortest the sampling period, the higher the model order.

5.2. Tuning of MPC

All simulations carried out for MPC tuning take into account 50 h of process operation. At the beginning of simulation the temperature set-point, T_{20}^{sp} , is increased by 5° C, to 20° C. It is required that the controller steers the process in such a way that the actual value of air temperature, T_{20} , reaches its new set-point as fast as possible. Furthermore, in the second part of the simulation it is assumed that the external temperature, T_{ext} , decreases by 30° C. In all experiments the same limits of the manipulated variable are included in the MPC optimisation problem

$$q^{\min} = 0, q^{\max} = 200 \quad (62)$$

The increments of the manipulated variable are not limited since the electric underfloor heating may be activated and deactivated very fast (in contrast to typical industrial application there are no slow actuators).

At first the prediction horizon, N , is selected. Fig. 6 depicts

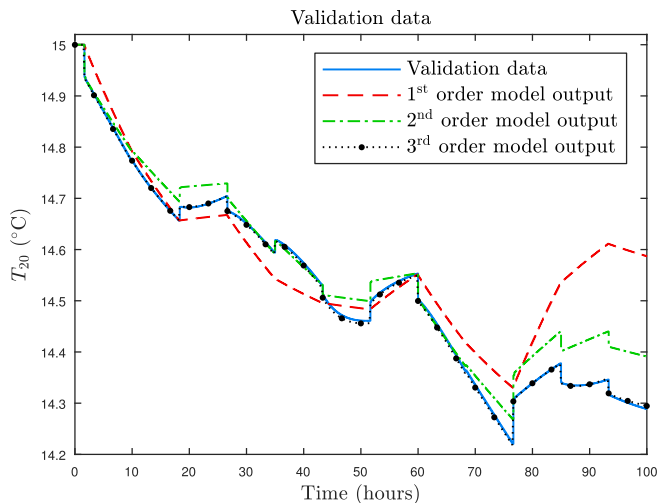


Fig. 5. The validation data vs. the output of the first-order, second-order and third-order models; $T_s = 600$ s.

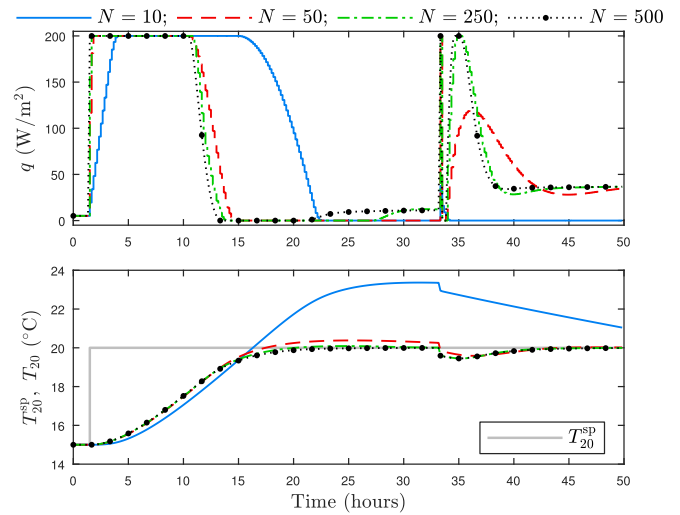


Fig. 6. Simulation results of the MPC algorithm for different prediction horizons N ; $N_u = 10$, $\lambda = 10^{-3}$, $T_s = 600$ s.

simulation results of the MPC algorithm for different prediction horizons $N = 10$, $N = 50$, $N = 250$ and $N = 500$, in all cases $N_u = 10$, $\lambda = 10^{-3}$. If not specified otherwise, the sampling time is $T_s = 600$ s and the third-order model (Table 1) is used in MPC. It is obvious that for too short prediction horizons the resulting control quality is low. As a reasonable compromise, the value $N = 250$ is chosen since lengthening of the horizon has a very small effect on control quality.

Next, the control horizon, N_u , is chosen. Fig. 7 depicts simulation results for $N_u = 1$, $N_u = 2$, $N_u = 5$, $N_u = 10$, in all cases $N = 250$, $\lambda = 10^{-3}$. In this case the value $N_u = 5$ is chosen since it results in good control quality, very close to that possible when the control horizon is doubled. Similarly to the prediction horizon, short control horizons result in slow output trajectories.

Next, the factor λ is tuned. Fig. 8 depicts simulation results for $\lambda = 10^{-1}$, $\lambda = 10^{-2}$, $\lambda = 10^{-3}$, $\lambda = 10^{-4}$, in all cases $N = 250$, $N_u = 5$. In order to obtain fast set-point tracking and compensation of the disturbance, the value $\lambda = 10^{-3}$ is finally chosen. Of course, the bigger the value of the parameter λ , the slower the trajectories of

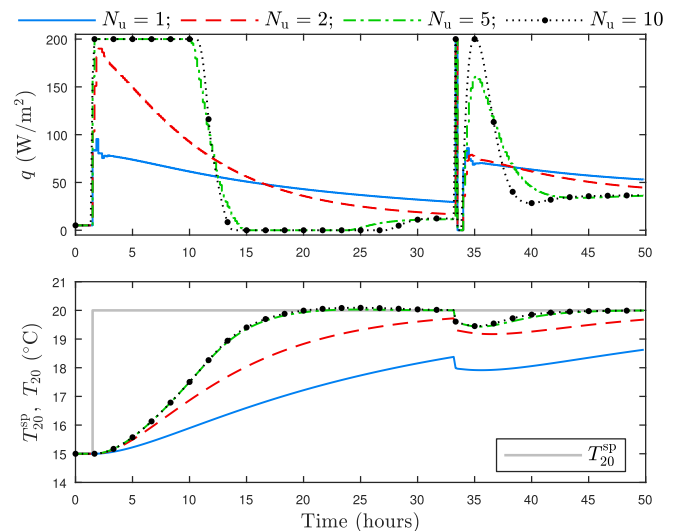


Fig. 7. Simulation results of the MPC algorithm for different control horizons N_u ; $N = 250$, $\lambda = 10^{-3}$, $T_s = 600$ s.

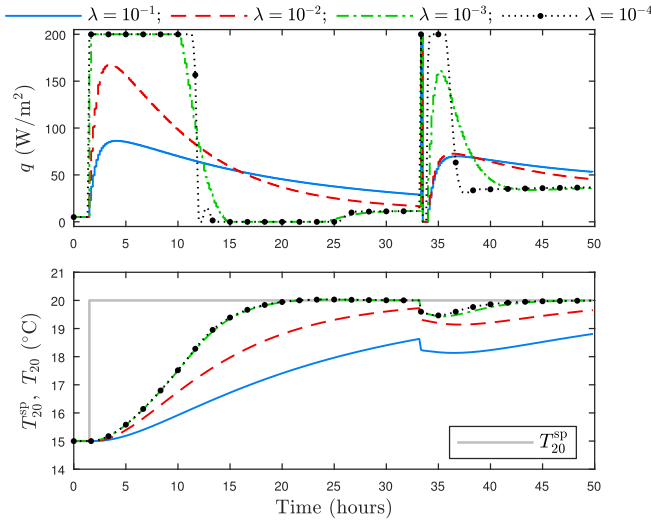


Fig. 8. Simulation results of the MPC algorithm for different values of the coefficient λ ; $N_u = 5$, $N = 250$, $T_s = 600$ s.

the process input and output variables.

It is an important question if and to what extent the value of the sampling period of the MPC algorithm, T_s , influences the possible quality of control. Fig. 9 depicts simulation results for two cases: $T_s = 600$ s and $T_s = 60$ s. In both cases $N_u = 5$, $\lambda = 10^{-3}$. For $T_s = 600$ s and $T_s = 60$ s, for prediction the third-order model and the fifth-order one are used, respectively (Table 1). One may note that the nominal prediction horizon, $N = 250$, selected for the long sampling period $T_s = 600$, should be lengthened when the sampling time is reduced, since the trajectories for the nominal prediction horizon are slow and characterized by long-lasting overshoot. Therefore, for the short sampling time $T_s = 60$ the prediction horizon $N = 1000$ is recommended. It is interesting to note the fact that when well tuned, the controllers with the sampling time $T_s = 600$ and $T_s = 60$ result in very similar process output trajectories. Fig. 10 depicts simulation results when $T_s = 600$ and $T_s = 300$. In both cases $N_u = 5$, $\lambda = 10^{-3}$. For $T_s = 600$ and $T_s = 300$, for prediction the third-order model and the fourth-order one are used, respectively (Table 1). One may note that for the sampling

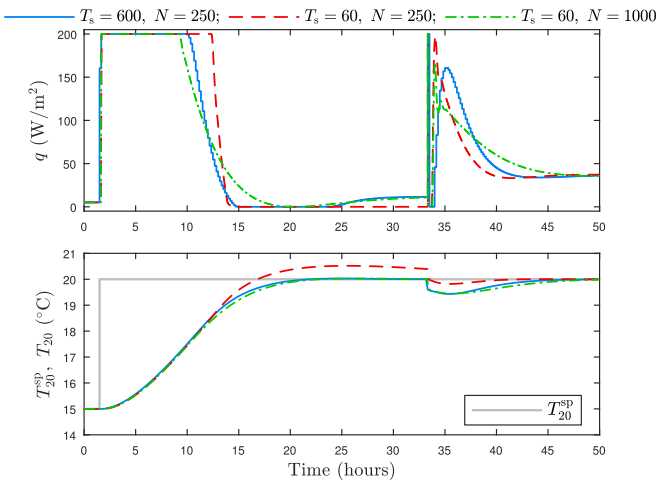


Fig. 9. Simulation results of the MPC algorithm for $T_s = 600$ s (with the nominal prediction horizon $N = 250$) and for $T_s = 60$ s (with prediction horizons $N = 250$ and $N = 1000$), $N_u = 5$, $\lambda = 10^{-3}$.

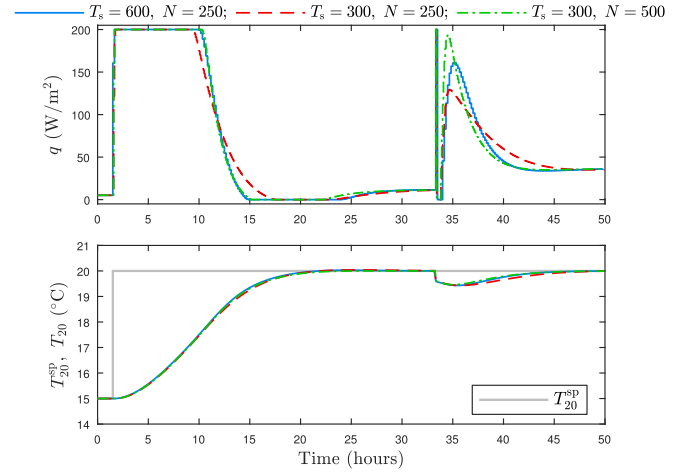


Fig. 10. Simulation results of the MPC algorithm for $T_s = 600$ s (with the nominal prediction horizon $N = 250$) and for $T_s = 300$ s (with prediction horizons $N = 250$ and $N = 500$), $N_u = 5$, $\lambda = 10^{-3}$.

period $T_s = 300$ the doubled prediction horizon ($N = 500$) results is slightly faster trajectory of the process output, but this difference is very small. It means that for $T_s = 300$ the nominal prediction horizon $N = 250$ may be used, the same as for $T_s = 600$. All things considered, the found prediction horizons are: $N = 250$ when $T_s = 300$ or $T_s = 600$, $N = 1000$ when $T_s = 60$. The control horizon and the weighting coefficient are independent of the sampling period: $N_u = 5$, $\lambda = 10^{-3}$.

5.3. Process modelling for energy optimisation

From the dynamic nonlinear model described in Section 2 one may easily obtain the steady-state nonlinear process description

$$0 = f(T_1, \dots, T_{20}, q, T_{\text{ext}}) \quad (63)$$

Recalling that the process output is the temperature T_{20} , it is also possible to find numerically the steady-state model $T_{20}(q, T_{\text{ext}})$, i.e. calculate the value of the temperature T_{20} for given values of the manipulated and disturbance signals, q and T_{ext} , respectively. Next, a simpler empirical steady-state model is obtained to be used in set-point optimisation. The simplest approach is to find a linear model (58). At first, the temperature T_{20} is calculated from the nonlinear model (63) for a mesh of points from the range $0 \text{ W/m}^2 \leq q \leq 200 \text{ W/m}^2$ and $-35 \text{ }^\circ\text{C} \leq T_{\text{ext}} \leq 35 \text{ }^\circ\text{C}$. Next, the model parameters, m_0 , m_1 and m_2 , are calculated using the least squares algorithm the objective of which is to minimise the squared differences between the model outputs and available data points. Although the rudimentary fundamental steady-state model (63) is nonlinear, the resulting linear model (58) approximates properties of the process very well as shown in Fig. 11. It means that the set-point optimiser solves on-line the quadratic optimisation problem (59), nonlinear optimisation is not necessary.

5.4. Energy optimisation

Fig. 12 depicts changes of the external temperature, T_{ext} , during the whole calendar year. The external temperature has been recorded in Kraków, Poland. The heating season lasts from October 1 till April 30. In simulations discussed in the following part of this work two configurations are considered:

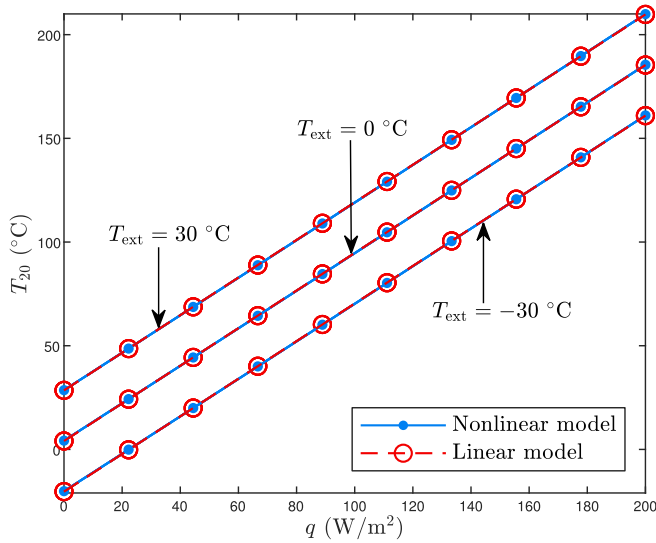


Fig. 11. Nonlinear steady-state model $T_{20}(q, T_{\text{ext}})$ vs. its linear approximation.

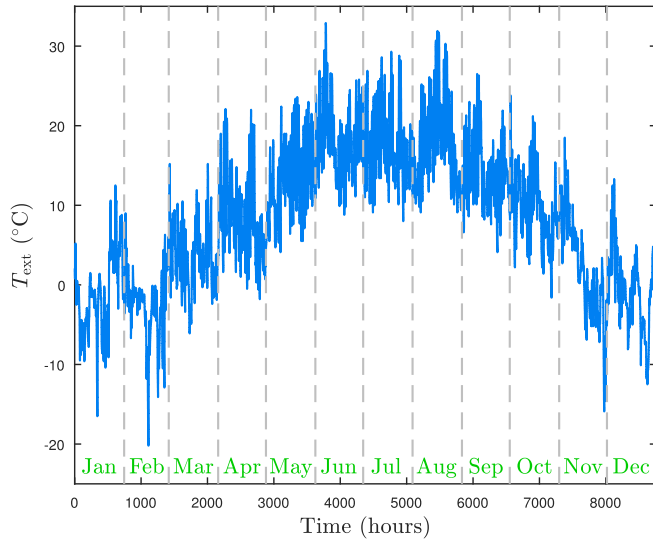


Fig. 12. Changes of the external temperature, T_{ext} , during the calendar year.

- The MPC algorithm with constant set-point $T_{20}^{\text{sp}} = 18.5^\circ\text{C}$ or $T_{20}^{\text{sp}} = 21^\circ\text{C}$. In the first case the constraints imposed on the predicted output variable are defined by Eq. (52), in the second case by Eq. (53). The output constraints are implemented as soft ones. In both cases the magnitude of the manipulated variable is constrained according to Eq. (62). Three sampling periods are considered: 60 s, 300 s, 600 s.
- The MPC algorithm with set-point optimisation. At each sampling instant the current value of the set-point is successively calculated on-line for the measured value of the disturbance signal. Two configurations of constraints imposed on the predicted output variable are used, i.e. defined by Eqs. (52) and (53), also implemented as soft ones. The limits of the manipulated variable and the sampling periods are the same as in the first configuration.

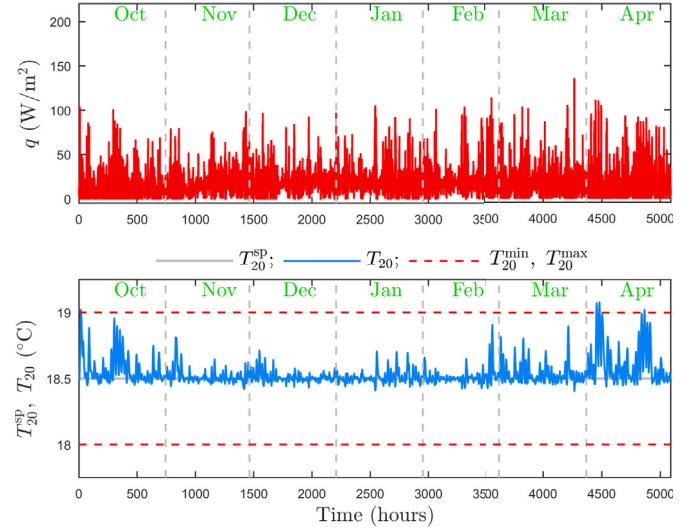


Fig. 13. Simulation results: MPC with constant set-point $T_{20}^{\text{sp}} = 18.5^\circ\text{C}$ and the sampling time $T_s = 60$ s, $T_{20}^{\text{min}} = 18^\circ\text{C}$, $T_{20}^{\text{max}} = 19^\circ\text{C}$.

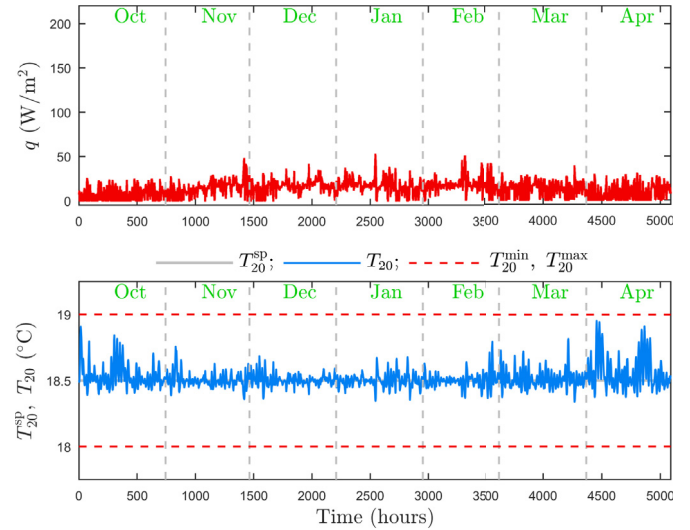


Fig. 14. Simulation results: MPC with constant set-point $T_{20}^{\text{sp}} = 18.5^\circ\text{C}$ and the sampling time $T_s = 300$ s, $T_{20}^{\text{min}} = 18^\circ\text{C}$, $T_{20}^{\text{max}} = 19^\circ\text{C}$.

$T_{20}^{\text{sp}} = 18.5^\circ\text{C}$ for the sampling periods 60 s, 300 s and 600 s, respectively. It is interesting to note that changes of the controlled variable, T_{20} , caused by variations of the disturbance (Fig. 12), are quite small. It means that the controller compensates fast changes of the disturbance. Generally, the constraints imposed on the predicted process output (Eq. (52)) are satisfied, the only exception is for the sampling period 60 s (small violations of the upper constraint are observed in October and April). The second important observation is the fact that taking into account the process output, for three compared sampling periods the process output trajectories are similar. However, the trajectories of the process input variable are different. In particular, for the shortest sampling period variability of the manipulated variable is bigger when compared with the results obtained for longer sampling times. Happily, this variability is not dangerous since in the considered control system there is no valve which moves physically, the manipulated variable only changes voltage of the electric heating foil. Fig. 16 shows simulation results for the constant set-point

Figs. 13–15 depict simulation results obtained when the process is controlled by the MPC algorithm with the constant set-point

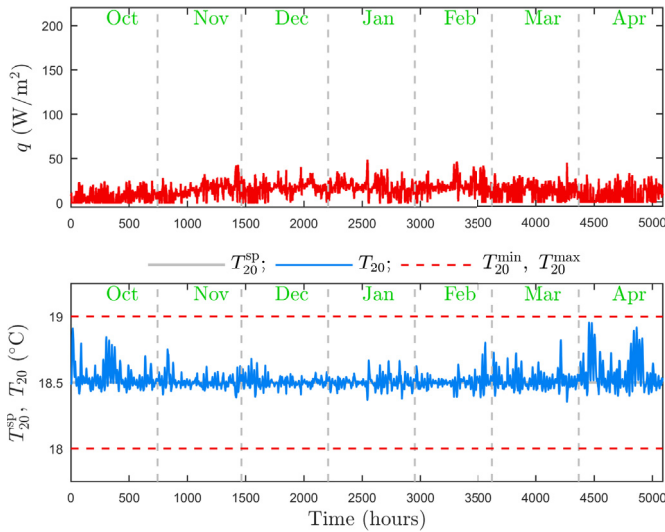


Fig. 15. Simulation results: MPC with constant set-point $T_{20}^{\text{sp}} = 18.5$ °C and the sampling time $T_s = 600$ s, $T_{20}^{\text{min}} = 18$ °C, $T_{20}^{\text{max}} = 19$ °C.

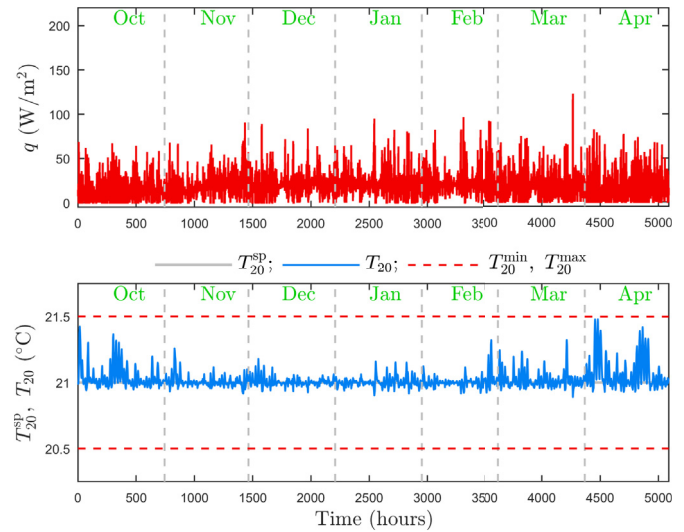


Fig. 16. Simulation results: MPC with constant set-point $T_{20}^{\text{sp}} = 21$ °C and the sampling time $T_s = 60$ s, $T_{20}^{\text{min}} = 20.5$ °C, $T_{20}^{\text{max}} = 21.5$ °C.

$T_{20}^{\text{sp}} = 21$ °C and the sampling period 60 s. Also in this case the MPC algorithm controls the process in such a way that variations of the controlled variable are small.

Table 2 compares the first configuration numerically. For each sampling period and for two possible set-points the amount of KWh used during the heating season and the resulting cost of heating (Eq. (54)) are specified. Additionally, the degree of violation of the hard output constraints is calculated from

$$E_{T_{20}} = \frac{1}{N_{\text{sim}}} \sum_{k=1}^{N_{\text{sim}}} \left[\left(\max(T_{20}^{\text{min}} - T_{20}(k), 0) \right) + \left(\max(T_{20}(k) - T_{20}^{\text{max}}, 0) \right) \right] \quad (64)$$

Interpretation of the coefficient $E_{T_{20}}$ is straightforward: when the hard constraints specified by Eq. (52) or Eq. (53) are satisfied, $E_{T_{20}} = 0$. Conversely, when they are violated, $E_{T_{20}}$ summarises absolute values of violations. From Table 2 three conclusions can be drawn. Firstly, energy consumption is lower by approx. 1430 KWh for the lower set-point, $T_{20}^{\text{sp}} = 18.5$ °C when compared with the set-point $T_{20}^{\text{sp}} = 21$ °C. Secondly, for the same set-point, changes of the sampling period have very small effect on energy used. Thirdly, because the set-point is between the output constraints, satisfaction of constraints is very good.

Figs. 17–19 depict simulation results obtained when the set-point for the MPC algorithm is repeatedly optimised on-line for the measured value of the disturbance for the sampling periods 60 s, 300 s and 600 s, respectively. The constraints imposed on the predicted output variable are defined by Eq. (52). In the majority of

sampling instants the optimiser finds that the minimal possible temperature should be taken as the set-point for the MPC controller. It may be observed that for the shortest sampling period changes of the set-point trajectory are followed faster when compared with medium and long sampling periods. Similarly to simulations with the MPC algorithm only, variability of the manipulated variable is significant for the shortest sampling period. Figs. 20 and 21 demonstrate simulation results of MPC with set-point optimisation for constraints of the output variable defined by Eq. (53), for the sampling period 60 s and 600 s, respectively. Also in this case during the majority of simulation scenario the optimiser chooses the minimal value of the output variable as the set-point and for the shorter sampling period variability of signals is bigger.

Table 3 compares the second configuration numerically for each sampling period and for two possible pairs of output constraints. Three observations may be made. Firstly, lowering the minimal and maximal values of the output constraints by 2.5 °C reduces the

Table 2

Simulation results: MPC with constant set-points T_{20}^{sp} for different sampling time T_s ; K_{total} is cost of heating, $E_{T_{20}}$ is degree of violation of the hard output constraints.

T_s (s)	T_{20}^{sp} (°C)	KWh used	K_{total} (EUR)	$E_{T_{20}}$
60	18.5	7019.80	912.57	9.85×10^{-5}
300	18.5	7009.04	911.17	0
600	18.5	7010.39	911.35	0
60	21	8449.81	1098.47	0
300	21	8444.82	1097.83	0
600	21	8446.17	1098.00	0

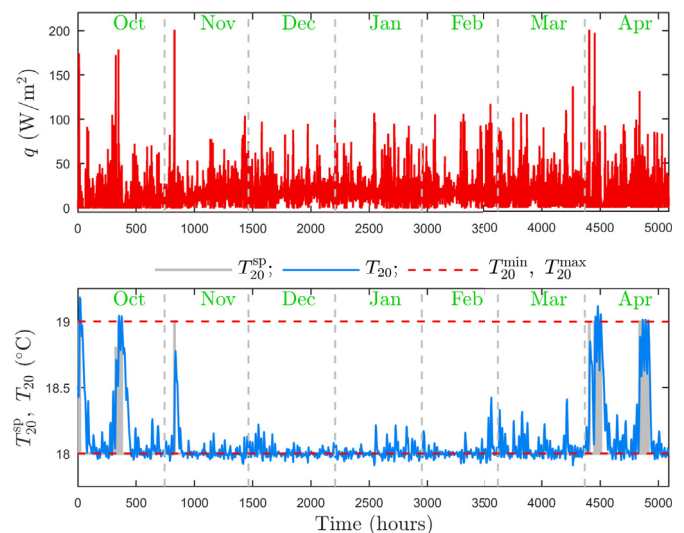


Fig. 17. Simulation results: MPC with set-point optimisation and the sampling time $T_s = 60$ s, $T_{20}^{\text{min}} = 18$ °C, $T_{20}^{\text{max}} = 19$ °C.

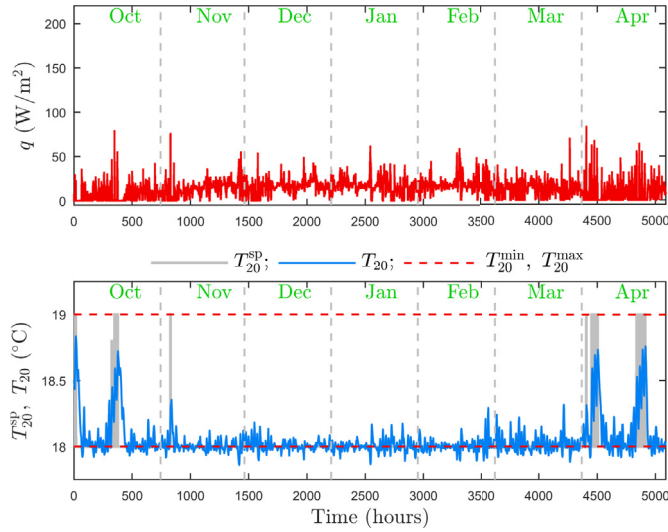


Fig. 18. Simulation results: MPC with set-point optimisation and the sampling time $T_s = 300$ s, $T_{20}^{\min} = 18$ °C, $T_{20}^{\max} = 19$ °C.

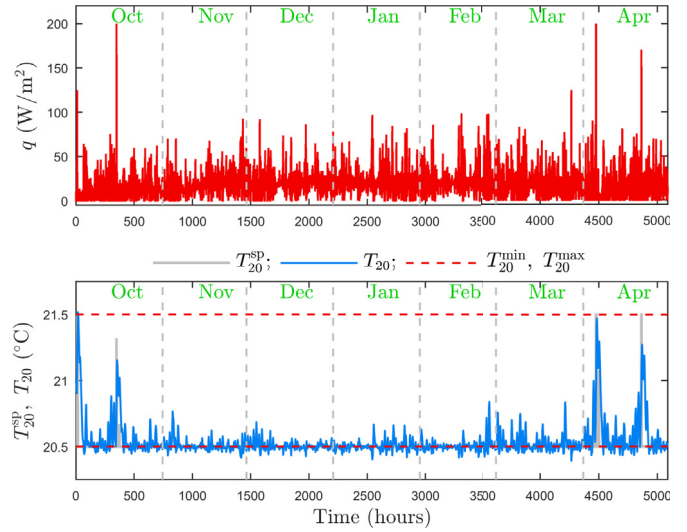


Fig. 20. Simulation results: MPC with set-point optimisation and the sampling time $T_s = 60$ s, $T_{20}^{\min} = 20.5$ °C, $T_{20}^{\max} = 21.5$ °C.

amount of energy used by approx. 1534 kWh (for the sampling period 60 s) or 1426 kWh (for the sampling periods 300 s i 600 s). Secondly, similarly to the first configuration when only the MPC algorithm with a constant set-point is used (Table 2), the sampling period has very small effect on the energy used. Thirdly, since the optimal set-point is typically equal to the minimal constraint of the process output, the degree of constraint violation is non-zero, although the obtained values are small.

Finally, it is necessary to compare two considered configurations: the MPC controller with a constant set-point (Table 2) and the MPC controller cooperating with the optimiser which repeatedly calculates the set-point (Table 3). For the sampling periods 300 s and 600 s introduction of the set-point optimiser makes it possible to reduce energy used up to approx. 300 kWh annually. For the sampling period 60 s the gains are 279 kWh and 174 kWh, for the output constraints defined by Eqs. (52) and (53), respectively.

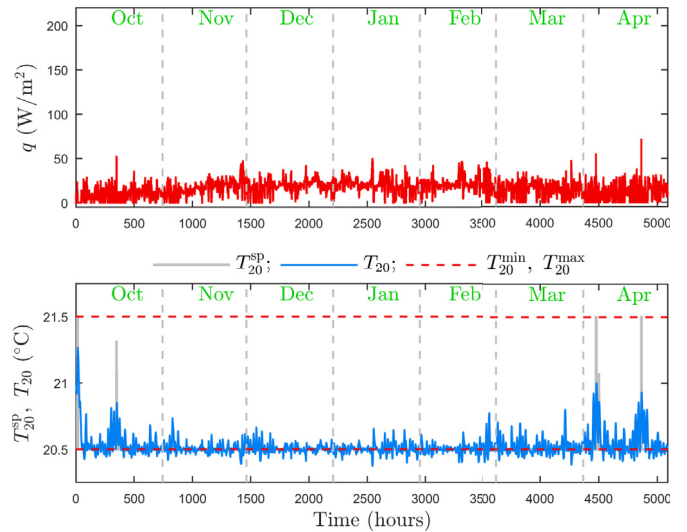


Fig. 21. Simulation results: MPC with set-point optimisation and the sampling time $T_s = 600$ s, $T_{20}^{\min} = 20.5$ °C, $T_{20}^{\max} = 21.5$ °C.

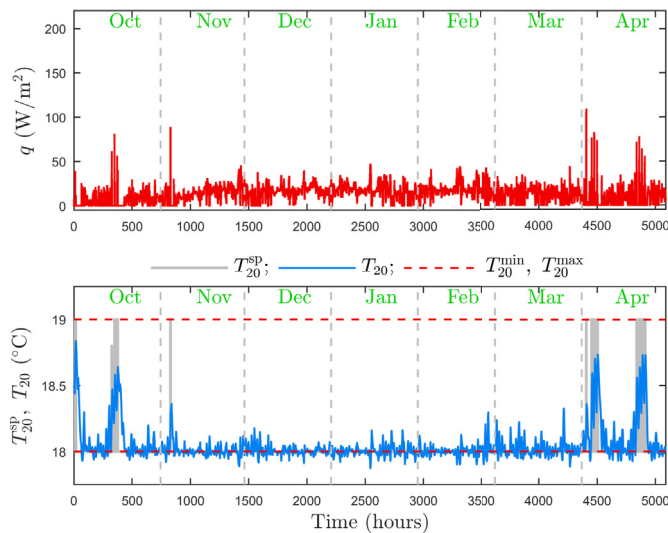


Fig. 19. Simulation results: MPC with set-point optimisation and the sampling time $T_s = 600$ s, $T_{20}^{\min} = 18$ °C, $T_{20}^{\max} = 19$ °C.

6. Conclusions

This work is concerned with a residential building with under-floor electric heating foils. At first, the first-principle mathematical model of the process is detailed. Next, two approaches to temperature control are developed and compared: MPC with a constant temperature set-point and MPC cooperating with the set-point optimiser which repeatedly calculates on-line the set-point in order to minimise the amount of energy used. The presented solution is computationally efficient, since in MPC and set-point optimisation classical quadratic optimisation is used on-line. It makes it possible to use for implementation relatively not complicated hardware platforms, advanced ones are not necessary [21–23].

Taking into account the obtained results, the following conclusions may be drawn:

Table 3Simulation results: MPC with set-point optimisation for different sampling time T_s ; K_{total} is cost of heating, $E_{T_{20}}$ is degree of violation of the hard output constraints.

T_s (s)	Soft output constraints	KWh used	K_{total} (EUR)	$E_{T_{20}}$
60	$T_{20}^{\min} = 18^\circ\text{C}$, $T_{20}^{\max} = 19^\circ\text{C}$	6741.17	876.35	7.36×10^{-3}
300	$T_{20}^{\min} = 18^\circ\text{C}$, $T_{20}^{\max} = 19^\circ\text{C}$	6718.59	873.42	1.13×10^{-2}
600	$T_{20}^{\min} = 18^\circ\text{C}$, $T_{20}^{\max} = 19^\circ\text{C}$	6717.93	873.33	9.66×10^{-3}
60	$T_{20}^{\min} = 20.5^\circ\text{C}$, $T_{20}^{\max} = 21.5^\circ\text{C}$	8275.83	1075.86	5.00×10^{-3}
300	$T_{20}^{\min} = 20.5^\circ\text{C}$, $T_{20}^{\max} = 21.5^\circ\text{C}$	8144.15	1058.74	1.26×10^{-2}
600	$T_{20}^{\min} = 20.5^\circ\text{C}$, $T_{20}^{\max} = 21.5^\circ\text{C}$	8143.50	1058.65	1.08×10^{-2}

1. In the first case a natural method to reduce energy used is to slightly reduce the temperature set-point. For example, reduction of the set-point from 21°C to 18.5°C leads to reducing the energy usage by approx. 1430 KWh during the heating season.
2. In the second case lowering the minimal and maximal values of the output constraints by 2.5°C case reduces the amount of energy used by approx. 1500 KWh.
3. Comparing the two considered control systems structures, introduction of the set-point optimiser makes it possible to further reduce energy used by approx. 300 KWh annually.

An interesting direction of future research is optimisation of energy usage taking into account variability of electricity price (different prices for days and nights). In such a case a dynamic model of the process must be used not only in MPC, but also in energy optimisation. Another interesting issue is to use weather forecasts for optimisation.

Declaration of interest

None.

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