

Statistical measures for proportional–integral–derivative control quality: Simulations and industrial data

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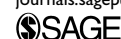
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Abstract

This article focuses on investigation of statistical approaches to the task of control performance assessment. Different statistical measures with Gaussian and non-Gaussian probabilistic distributions are taken into consideration. Analysis starts with the observations for simulated proportional–integral–derivative control error histograms followed by its statistical investigation using selected probabilistic distribution functions. Simulation experiments are followed by the analysis of control data originating from real industrial loops. Shadowing effect of long-tail control error histograms is identified, as it may significantly disable proper loop quality assessment. Results show that non-Gaussian approach with Cauchy or α -stable distributions seems to be reasonable assessment alternative in case of disturbances existing in industrial processes.

Keywords

Controller performance assessment, proportional–integral–derivative control, non-Gaussian distributions, α -stable probabilistic distribution function, industrial data

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Introduction

Assessments of hundreds industrial proportional–integral–derivative (PID) loops have formed the rationale for that work. Control quality is one of the key elements constituting overall installation performance. It enables to reach and sustain maximum achievable benefits of the process. Thus, any performance analysis or feasibility study should include control quality assessment. Applicability of Control Performance Assessment (CPA) requires introduction of commonly understood measures that allows to quantify observations. There are many indexes, which were quickly verified by industry. Basic statistics, like mean and variance, are mainly used in commercial performance studies. This work tries to investigate their applicability in case of disturbances that are frequently witnessed in industry.

CPA subject exists for years. It is important, as large number of control loops performs unsatisfactory, with 60% featuring poor tuning and 85% facing improper structure.¹ Research in that area started in 60s and grew up with introduction of minimum variance approach.² After that the rapid development started. As well-designed control system brings significant benefits,

supporting monitoring tools start to play important role. They become an inevitable element for good practice. They are required not only for single element or cascaded PID loops (being majority of industrial controls) but also for advanced process control (APC).³

Commercial assessment mostly uses basic approach calculating mean and standard deviation for variables, sometimes followed by higher order statistics.⁴ Often variable histograms are used. It has been noticed that approach with normal distribution statistics has shortcuts. Industrial data histograms hardly meet Gaussian properties. It has been shown that only minority of industrial control loops has histograms similar to normal distribution.⁵ Real processes are mostly nonlinear, cross-coupled, time varying and complex systems. They

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are exposed to numerous external and internal disturbances of various origins. They are affected by varying delays. Actuators frequently malfunction and sensors might be de-calibrated. Finally, there is also a human factor, like manual interventions or system tuning.

Although there exist many approaches, always a need for a robust solution well fitted to industrial conditions. This article tries to challenge that subject. It focuses on two aspects: which statistical approach is better in industrial conditions and how it is affected by disturbances.

Various probabilistic density functions are investigated as potential sources of new statistical factors that might be useful in the task of CPA. They are confronted with industrial conditions. Effects of the disturbances on control performance are investigated. It is shown that some of these coefficients are robust to the impeding disturbances being potential candidates as they allow reliable assessment. Procedure for the control loop diagnostic will be proposed to conclude the work.

This article starts with subject formulation illustrated by standard and statistical measures. Main contribution consists of data analysis for different PID configurations (P, PI, PID) exposed to various disturbances. Simulations are followed by real data review. Final section concludes the article with discussion and presentation of open issues.

Control performance assessment

Control engineer often uses his own approach to evaluate loop quality. It is frequently unique. Thus, the need for standardization and benchmarking arises. First reported control loop performance assessment was done in 1967⁶ for paper machine. Research has accelerated in 1989 with introduction of performance index based on minimum variance. CPA covers now various aspects of control, like nonlinearities,⁷ large-scale systems,⁸ predictive controllers,⁹ single input–single output (SISO) and multiple input–multiple output (MIMO) control structures,¹⁰ valve stiction,¹¹ oscillations¹² and others. We may determine six types:

1. Indexes based on step response—delay, undershoot, rising, maximum, settling time, overshoot and peak value;
2. Time measures—mean square error (MSE), integral absolute error (IAE), amplitude index (AMP);
3. Frequency domain methods, like with the use of the Fourier transform,¹³ sensitivity function¹⁴ or the reference to disturbance ratio (RDR) index;¹⁵
4. Minimum variance indexes,
5. Statistical measures of various PDFs—standard deviation, variance, skewness, kurtosis, scale and shape factors;
6. Alternative indexes, like fractal,^{16,17} multi-persistence⁵ or entropy measures;¹⁸

7. Business KPIs frequently expressed in monetary units.

This article presents comparison of two main approaches, that is, time series–based methods followed by statistical factors. It is also important to choose the appropriate loop signal for further analysis. One might use controlled variable (CV), control error (difference of setpoint and process variable (PV)) or controller output. Various authors frequently use CV. Control error signal is selected in the considered research as it is independent on operating points and should fluctuate around zero. Thus, there is no need for detrending.

Time domain methods and statistical measures are the most popular approaches used in practical applications. They are relatively simple, well understood by plant personnel and have direct technical meaning, often with clear reference to economical aspects. This is why these methods are mainly considered in the article, and thus, they are presented below.

Time domain methods

Step test measures (area, output, idle or *R*-index),¹⁹ although being very informative, have limited applicability. Industry hardly agrees for dedicated experiments or any artificial disturbance to normal operation. It is mostly because of safety and economical issues. Due to unanswered question, how to calculate these responses from normal operation data, they cannot be used. However, integral indexes are extremely popular. They use time series data from normal operation, which is easily available. MSE, IAE and AMP are further considered.

Statistical indexes

Factors of normal distribution form the most popular key performance indicators (KPIs). Mean value μ reflects variable average value, while standard deviation σ informs about signal variability. These measures are frequently followed by higher order statistics like skewness or kurtosis.

Importance of Gaussian measures is unquestionable. Majority of researchers and control engineers use them. However, we have to be aware that they are valid, while the signals have Gaussian properties. It was shown²⁰ that for large sets of industrial control error histograms only $\approx 6\%$ has normal distribution. Majority has fat tails. Mostly Lévy α -stable distribution is the best fitted (more than 60%), while the rest holds Cauchy probability density function (PDF) properties. To investigate potential of the statistical approach, the following functions will be considered: Cauchy, Lévy α -stable, Laplace and generalized extreme value (GEV).

Cauchy PDF is an example of the fat-tail distribution. The shape for values further from mean does not decay so fast, as it is with normal distribution. It is symmetric function. Its two parameters have similar

meaning to normal PDF (equation (1)). Location factor $\delta \in \mathbb{R}$ informs about distribution position, while scale $\gamma > 0$ reflects signal variability

$$PDF_{\delta, \gamma}(t) = \frac{1}{\pi\gamma} \left(\frac{\gamma^2}{(t - \delta)^2 + \gamma^2} \right) \quad (1)$$

Lévy α -stable distribution also belongs to the family of stable distributions. It has more degrees of freedom (equation (2)) as it is parametrized by four parameters

$$PDF_{\alpha, \beta, \delta, \gamma}(t) = \exp\{i\delta t - |\gamma t|^\alpha (1 - i\beta l(t))\} \quad (2)$$

$$l(t) = \begin{cases} \operatorname{sgn}(t) \operatorname{tg}\left(\frac{\pi\alpha}{2}\right) & \text{for } \alpha \neq 1 \\ -\operatorname{sgn}(t) \frac{2}{\pi} \ln|t| & \text{for } \alpha = 1 \end{cases}$$

$0 < \alpha \leq 2$ is called *stability index*, $|\beta| \leq 1$ is *skewness parameter*, $\delta \in \mathbb{R}$ is distribution *location* and $\gamma > 0$ is *scale*.

Stability parameter α is responsible for shape. Location δ keeps information about function position. In addition we have two more shape parameters. β informs about function skewness, while scaling γ is responsible for the distribution broadness (similarly to γ parameter of Cauchy PDF). There may be different combinations of the parameters. $\alpha = 2$ reflects independent realizations, especially when $\alpha = 2$, $\beta = 0$, $\gamma = 1$ and $\delta = 1$ we get exact normal distribution equation.

Laplace distribution is also called double exponential. It is symmetrical and forms a function of a difference between two independent variables with identical exponential distributions. Its PDF is given by formula (3)

$$PDF_{\mu, b} = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}} \quad (3)$$

where $\mu \in \mathbb{R}$ is *location factor* and $b > 0$ is *scale parameter*. Its shape decays exponentially and depends on parameter b .

GEV distribution is a family of continuous PDFs developed within extreme value theory to combine properties of various distributions (4). Parameters are

not exactly mean and standard deviation, which are calculated with equations (5) and (6)

$$PDF_{\mu, \sigma, \xi} = \frac{1}{\sigma} t(x)^{\xi+1} e^{-t(x)} \quad (4)$$

$$t(x) = \begin{cases} \left(1 + \left(\frac{x-\mu}{\sigma}\right)\xi\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ e^{-\frac{x-\mu}{\sigma}} & \text{if } \xi = 0 \end{cases}$$

$\mu \in \mathbb{R}$ is *location*, $\sigma > 0$ is *scale* and $\xi \in \mathbb{R}$ is *shape factor*

$$\text{mean}_{GEV} = \mu - \frac{\sigma}{\xi} + \frac{\sigma}{\xi} \cdot \Gamma(1 - \xi) \quad (5)$$

$$\text{stdev}_{GEV} = \sqrt{\frac{\sigma^2}{\xi^2} (\Gamma(1 - 2\xi) - \Gamma^2(1 - \xi))} \quad (6)$$

One may use location μ or mean value for steady-state error detection. Factors σ and ξ are responsible for shape and standard deviation as well.

There are effective methods²¹ to perform PDF fitting to histogram and estimate density function parameters. Cauchy, Laplace and GEV distribution parameter estimation use maximum likelihood estimation implemented in Octave successive quadratic programming solver. Stable PDF fitting uses Koutrouvelis²² regression.

Simulations

Simulation experiment is performed in the environment sketched in Figure 1. SISO loop uses PID controller with linear process described by transfer function $G(s)$. Base parallel PID structure is used.

Two cases are tested: without disturbance added before the plant (only with output noise) and disturbed by filtered Cauchy signal (zero mean and scale $\gamma = 0.2$) to reflect the disturbances frequently observed in the process industry. Setpoint is shaped as rectangular wave of period 100 and length of 2000 samples.

Four processes are considered. At first, optimal tuning is found for them. Standard Octave function, which minimizes weighted performance index consisting of integral time absolute error (ITAE) criterion (weighting

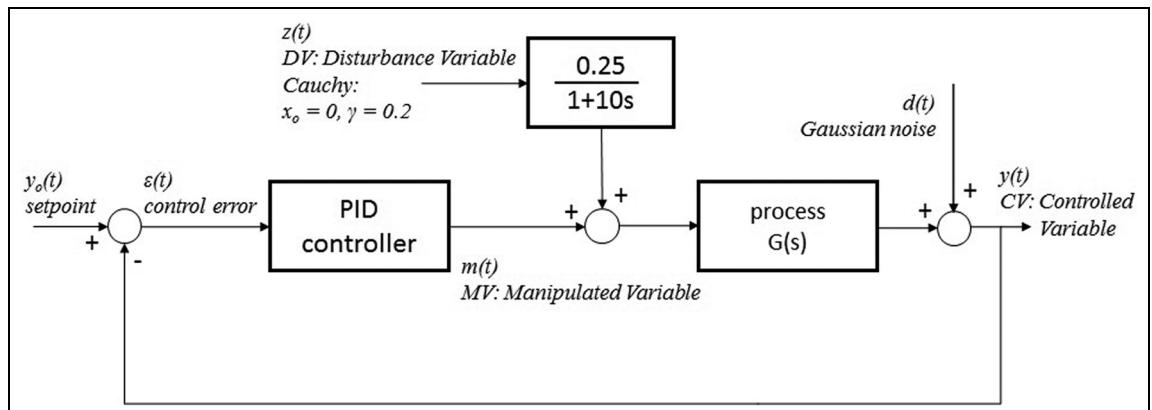


Figure 1. Closed-loop simulation environment.

Table 1. Optimal tuning parameters for selected processes.

	k_p	T_i	T_d	κ
G_0	0.809	13.767	2.861	1.80%
G_1	0.017	2.448	0.866	7.56%
G_2	0.080	0.282	3.488	3.51%
G_3	1.056	2.462	0.734	0.37%

factor $\mu_1 = 1$), max overshoot (weighting factor $\mu_2 = 10$) and sensitivity (weighting factor $\mu_3 = 20$).²³ Their optimal tuning is presented in Table 1.

1. Transfer function with two time constants and delay

$$G_0(s) = \frac{2}{(3s+1)(10s+1)}e^{-4s} \quad (7)$$

2. Transfer function with fast and slow elements

$$G_1(s) = \frac{100}{(s+2)^2} \left(\frac{1}{s+1} + \frac{0.05}{s+0.05} \right) \quad (8)$$

3. Oscillatory transfer function

$$G_2(s) = \frac{1}{(s+1)(s^2+0.2s+1)} \quad (9)$$

4. Non-minimum phase transfer function

$$G_3(s) = \frac{1-0.2s}{(3s+1)^3} \quad (10)$$

At first process, $G_0(s)$ is thoroughly analyzed. Next, the other ones are compared to illustrate the observations. Five scenarios are investigated:

S1—Proportional P control;

S2—PI controller;

S3—PID with k_p and T_i varying for constant $T_d = 2.861$;

S4—PID with $T_i = 13.767$ and varying k_p and T_d ;

S5—PID with $k_p = 0.809$ and varying T_i and T_d .

P controller—S1

Proportional loop reaches stability edge for $k_p > 2.0$. Statistical tests reject normality hypothesis in all simulations. Control is subject to non-zero steady-state error. It was successfully detected for all statistical factors representing position. Curve is similar to the theoretical relation $1/(1+K)$ (Figure 2). Analysis is repeated for disturbed loop data (Figure 3). Steady-state error detection is worse, especially with GEV. Fat-tail distributions perform the best.

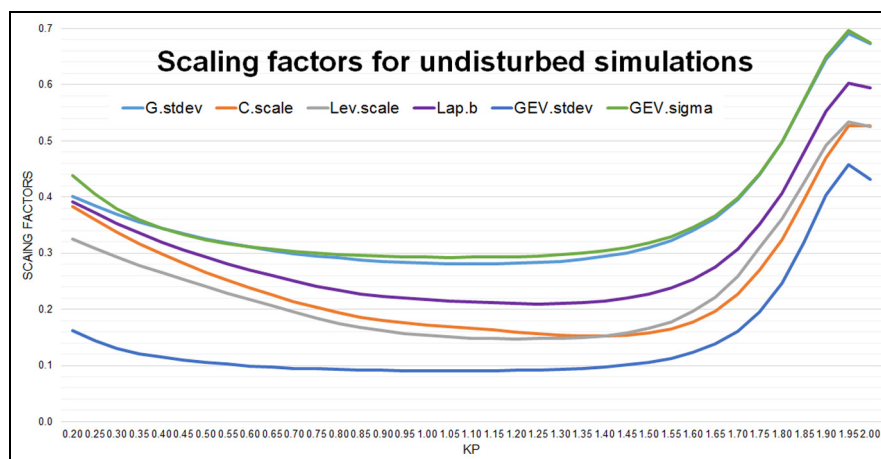
Second group of measures uses distribution scales. They should reflect control quality (variability). Unfortunately, ideal value of the measures is unknown and smaller value means better control (lower fluctuations).

Relation of scaling factors to k_p is evaluated; however, disturbances often disable proper detection. This phenomenon is called disturbance shadowing effect. Although in undisturbed cases detection (existence of single minimum) works, disturbances effectively disable it. Only fat-tail distributions perform properly. Values of k_p together with loop performance are sketched in Table 2. We see that detection prefers higher overshoot.

Indexes AMP, IAE (Figure 4) and MSE are verified. AMP index is the least informative as it is highly sensitive to maximum data peaks and its application is limited. MSE is of character similar to IAE, but it more deteriorates with disturbances. It only works properly in undisturbed case. IAE is more robust. MSE for step response selects $k_p = 1.35$ and on undisturbed data smaller gain $k_p = 1.20$. IAE points out $k_p = 1.15$ and $k_p = 1.20$. IAE favors smaller overshoot.

PI controller—S2

Analysis of PI case starts with location detection. No steady-state error exists so all mean factors should be

**Figure 2.** Detection with scale parameters for undisturbed P control.

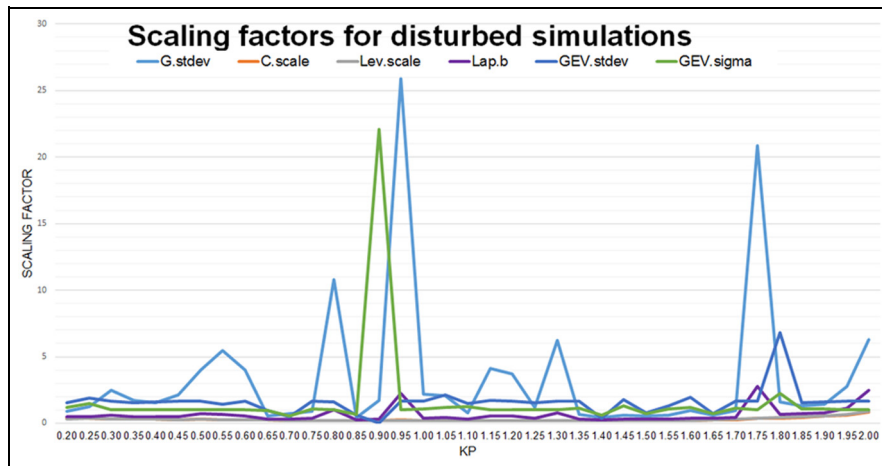


Figure 3. Detection with scale parameters for disturbed P control.

Table 2. Detection of performance with scaling factors for proportional P control.

	$G.\sigma$	$C.\gamma$	$Lev.\gamma$	$Lap.b$	$GEV.stdev$	$GEV.\sigma$
nDist	0.2810	0.153	0.148	0.210	0.090	0.294
k_p	1.10	1.40	1.20	1.25	1.05	1.05
κ	48.0%	69.5%	54.5%	58.1%	44.9%	44.9%
Dist	N/A	0.176	0.175	N/A	N/A	N/A
k_p	N/A	1.10	1.10	N/A	N/A	N/A
κ		48.0%	48.0%			

GEV: generalized extreme value.

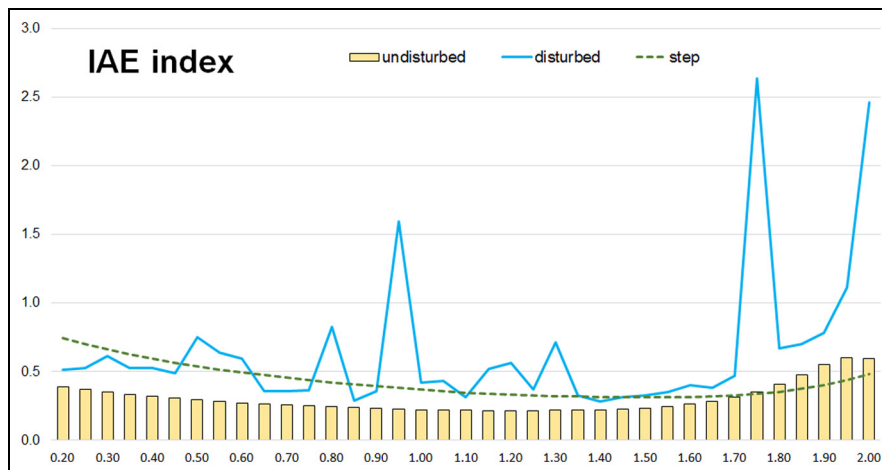


Figure 4. Detection with IAE index for P control.

close to zero. Undisturbed data are fully consistent with expectations. But disturbed operation is not. Especially results of GEV distribution are fully disappointing and as such they do not allow any kind of detection. Estimates for α -stable location parameter are useless. Almost all distributions have problems with proper detection of control error mean zero value. It is due to incidental values scattering far away from zero. Only Cauchy position factor μ is consistent. Laplace follows Gaussian values. GEV factors are not informative.

Almost all distributions have problems with detection of control error mean value, which should be zero. It is due to incidental values scattering far away from zero. Only Cauchy position factor μ is consistent. Laplace follows Gaussian values. GEV factors are non informative.

It is expected that scale and shape parameters can select single solution. The results are plotted with two-dimensional (2D) contour plots, that is, index versus k_p and T_i . Undisturbed surface for normal standard

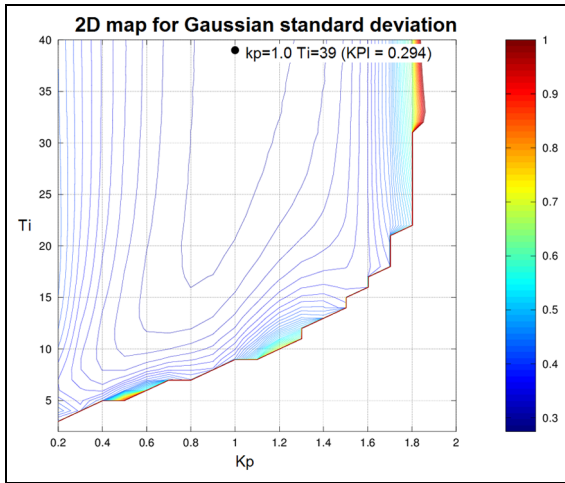


Figure 5. Detection for undisturbed PI with normal standard deviation σ .

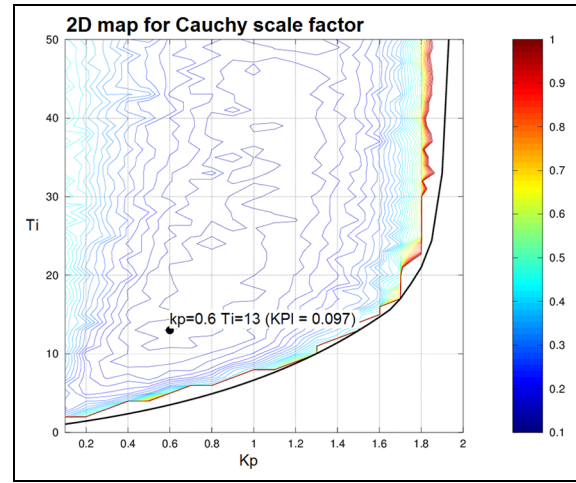


Figure 8. Detection for disturbed PI with Cauchy scaling parameter γ .

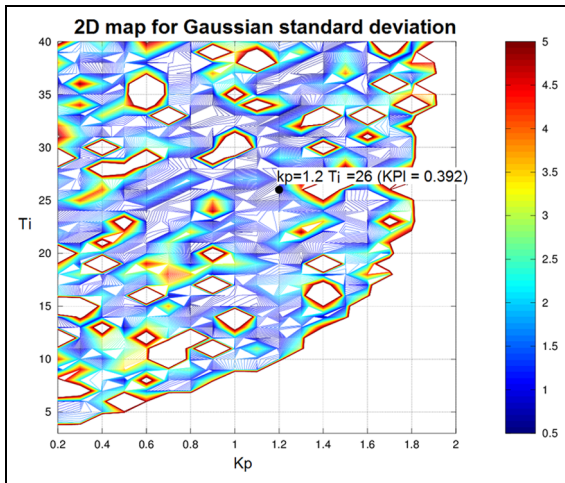


Figure 6. Detection for disturbed PI with normal standard deviation σ .

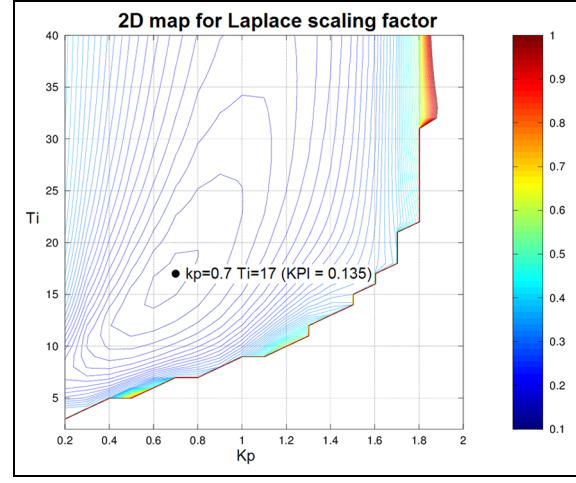


Figure 9. Detection for undisturbed PI with Laplace b factor.

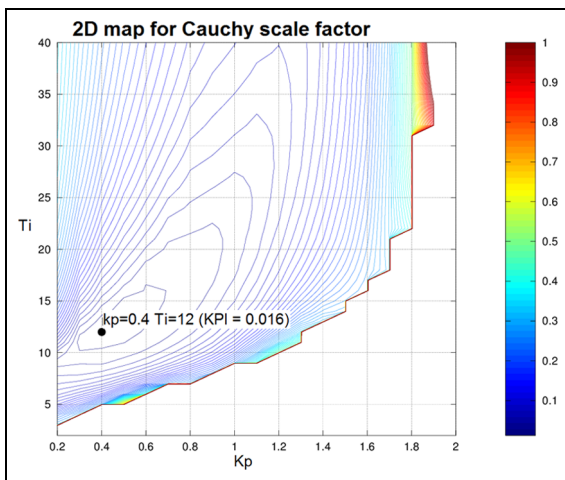


Figure 7. Detection for undisturbed PI with Cauchy scaling parameter γ .

deviation is smooth with clear minimum at $k_p = 1.0$, $T_i = 39.0$ and $\sigma = 0.294$ (Figure 5). It represents sluggish response with small overshoot. Disturbed plot is ragged with numerous peaks (Figure 6). The best solution lies in a probably random valley with $k_p = 1.2$, $T_i = 26$ and $\sigma = 0.392$. It is shadowing effect.

The best settings for Cauchy γ without disturbances are $k_p = 0.4$, $T_i = 12.0$ with $\gamma = 0.016$ (Figure 7). We get similar solution with disturbances, that is, $k_p = 0.6$, $T_i = 13.0$ ($\gamma = 0.097$) (Figure 8). It is worth to highlight that detection is not spoiled by disturbances. $k_p = 0.4$, $T_i = 12.0$ with $\gamma = 0.017$ are detected as optimal solution for Lévy scaling with no disturbances. We get relatively close solution $k_p = 0.6$, $T_i = 13.0$ ($\gamma = 0.097$) in disturbed case. Cauchy and Lévy give very similar contours and work properly.

Results for Laplace and GEV are very similar to normal PDF (Figure 9). Minimum point for undisturbed case is $k_p = 1.0$, $T_i = 40.0$ ($KPI = 0.108$). We

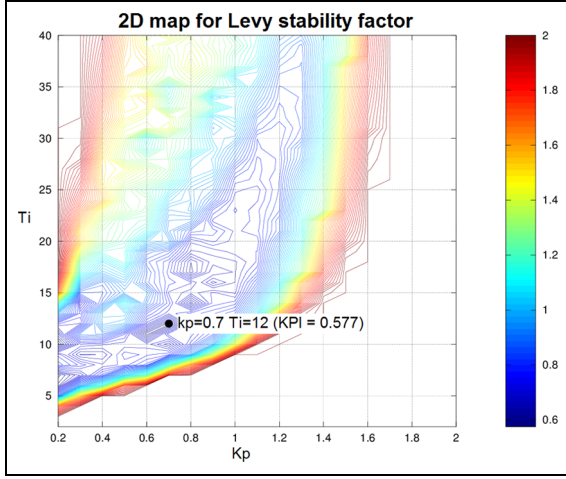


Figure 10. Detection for undisturbed PI with Lévy stability α .

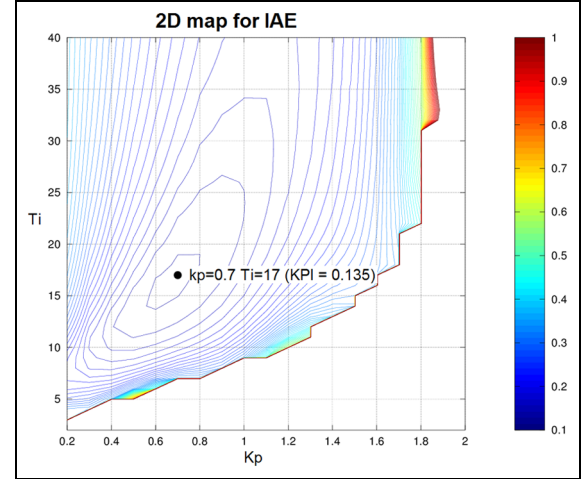


Figure 13. IAE index with undisturbed PI control.

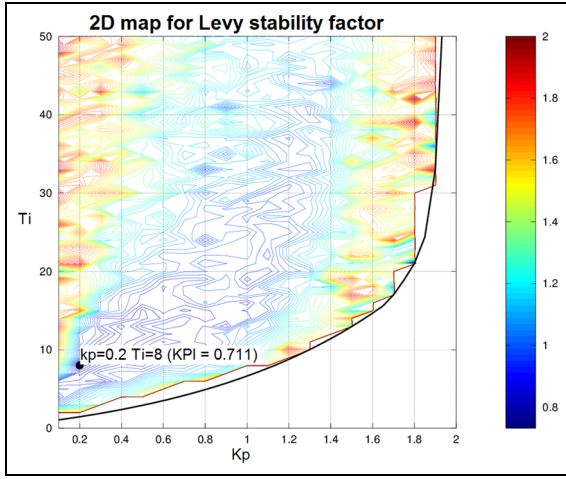


Figure 11. Lévy stability α detection for disturbed PI.

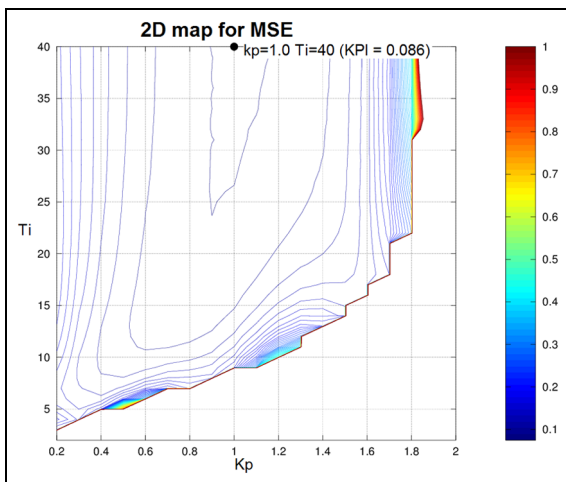


Figure 12. MSE index with undisturbed PI control.

get different solution in case of disturbances: $k_p = 1.2$, $T_i = 26.0$ and $KPI = 0.245$. Surface shape is very ragged.

Analysis of shaping factors for PI control has shown that they may be screened by fat-tail disturbances. Thus, only simple statistics are calculated for Lévy skewness parameter β . Detection is disturbed although is close to zero.

For undisturbed case stability factor α detects point $k_p = 0.7$, $T_i = 12.0$ ($KPI = 0.577$) (Figure 10). We get solution: $k_p = 0.2$, $T_i = 8.0$ with $KPI = 0.711$ in case of disturbances (Figure 11). Although the landscape is more ragged in disturbed case, minimum is visible.

Integral indexes find different solutions. MSE selects point $k_p = 0.9$, $T_i = 20.0$ with $KPI = 0.087$, while IAE $k_p = 0.6$, $T_i = 14.0$ and $KPI = 0.131$. MSE finds sluggish solution $k_p = 1.0$, $T_i = 40.0$ with $MSE = 0.086$ (Figure 12) for undisturbed case. Disturbed case delivers very ragged surface with settings $k_p = 1.0$, $T_i = 36.0$ ($MSE = 0.155$). IAE works better. It detects $k_p = 0.7$, $T_i = 17.0$ and $IAE = 0.135$ (Figure 13) for undisturbed case. With disturbances, it shows less smooth surface and optimal tuning $k_p = 1.0$, $T_i = 36.0$ ($IAE = 0.226$) being exactly the same as for MSE. AMP is not considered.

Methods find various, sometimes inconsistent, solutions. Overshoot κ (Table 3) is compared as a benchmark measure. We observe the following:

1. Loop disturbances, effectively shadow controller performance and impede proper detection, in most of cases.
2. MSE favors more aggressive solutions than IAE.
3. Gaussian standard deviation does not give good results.
4. GEV distribution is not appropriate at all.
5. Laplace gives the same results as normal PDF in undisturbed case, while with disturbances it is more robust.
6. Lévy factor α gives better detection with disturbances.
7. Assessment with scaling γ is not biased.

Table 3. Comparison of best solutions with different indexes.

		MSE	IAE	G. σ	C. γ	Lev. α	Lev. γ	Lap.b	GEV.std
undist	k_p	1	0.7	1	0.4	0.7	0.4	0.7	1
	T_i	40	17	39	12	12	12	17	40
	κ	16.0%	9.3%	16.1%	0.0%	24.5%	0.0%	9.3%	16.0%
dist	k_p	1	1	1.2	0.6	0.2	0.6	0.8	1.2
	T_i	36	36	26	13	8	13	18	26
	κ	16.7%	16.7%	35.3%	11.8%	2.0%	11.8%	15.7%	35.3%
step	k_p	0.9	0.6						
	T_i	20	14						
	κ	20.4%	8.5%						

Bold values describe the best solutions closest to the optimal solution. MSE: mean square error; IAE: integral absolute error; GEV: generalized extreme value.

Table 4. PID controller scenario S3 undisturbed case.

	G. σ	C. γ	Lev. α	Lev. γ	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.258	0.011	0.461	0.013	0.094	0.094	0.094	0.067	0.085	0.065
k_p	1.1	0.8	0.3	0.8	0.9	1.2	0.9	1.1	0.9	1.1
T_i	16.0	14.0	11.0	14.0	14.0	40.0	14.0	16.0	14.0	16.0
κ	17.3%	0.7%	0.9%	0.7%	6.8%	12.3%	6.8%	17.3%	6.8%	17.3%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

Table 5. PID controller scenario S3 disturbed case.

	G. σ	C. γ	Lev. α	Lev. γ	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.327	0.066	0.796	0.067	0.175	0.190	0.175	0.107	0.085	0.065
k_p	1.4	1.2	1.1	1.2	1.4	1.4	1.4	1.4	0.9	1.1
T_i	19.0	17.0	10.0	17.0	19.0	32.0	19.0	19.0	14.0	16.0
κ	35.7%	23.2%	32.1%	23.2%	35.7%	28.8%	35.7%	35.7%	6.8%	17.3%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

PID controller

PID scenario S3 starts with location detection. Results are very similar to PI case. Only Cauchy distribution mean is robust, although observation of average and median of estimated parameters shows that almost all distributions except GEV and α -stable perform accurately.

Detection results with scaling factors for both runs, that is, for undisturbed (Table 4) and disturbed cases (Table 5) are evaluated. Contour plots for selected indexes are analyzed. In fact, scenario does not differ much from PI (S2) case. Most of the disturbed simulations show ragged surfaces. The most significant observation is that we do not witness disturbance shadowing effect in case of Cauchy.

Laplace scaling b works somehow between Gauss and Cauchy. In undisturbed case, it is close to γ with slightly different surface and for disturbed case, it is close to σ . Disturbances effectively screen all effects for MSE and IAE. The shapes differ from each other. MSE is less decisive as minimum valley is flatter.

Selected tuning is more aggressive giving higher controller gain. Flat region favors solutions with high integration time T_i . IAE is decisive (*solution valley* is steeper). Identified tuning is shifted toward lower gain. Similar behavior is observed for PI control in step and normal operation (with very similar shapes as for PI).

We observe that MSE index delivers similar surface as normal standard deviation, while absolute error index results in surfaces close to γ of Cauchy and Laplace b parameter.

Analysis for PID scenario S4 starts with location identification. As previously, α -stable and GEV PDFs do not perform well. Quality assessment with scaling factors detection is done for undisturbed (Table 6) and disturbed (Table 7) cases.

Observations for scaling factors give similar results as previously. Only Cauchy distribution (Figure 14) is not shadowed and shows smooth surface in disturbed case. Laplace behaves as previously. Character of integral indexes is the same. MSE selects similar solutions

Table 6. PID controller scenario S4 undisturbed case.

	$G.\sigma$	$C.\gamma$	$Lev.\alpha$	$Lev.\gamma$	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.256	0.011	0.516	0.012	0.091	0.095	0.091	0.066	0.084	0.064
k_p	1.1	0.8	0.4	0.8	0.9	1.1	0.9	1.1	0.9	1.1
T_d	3.6	2.8	2.6	2.8	3.4	3.6	3.4	3.6	3.4	3.8
κ	20.7%	1.7%	0.0%	1.7%	5.1%	20.7%	5.1%	20.7%	5.1%	21.2%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

Table 7. PID controller scenario S4 disturbed case.

	$G.\sigma$	$C.\gamma$	$Lev.\alpha$	$Lev.\gamma$	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.321	0.066	0.775	0.068	0.181	0.132	0.181	0.103	0.084	0.064
k_p	1.3	1.0	0.7	1.1	1.3	1.3	1.3	1.3	0.9	1.1
T_d	1.8	3.6	1.8	3.6	4.2	1.8	4.2	1.8	3.4	3.8
κ	41.2%	12.7%	4.1%	20.7%	40.8%	41.2%	40.8%	41.2%	5.1%	21.2%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

Table 8. PID controller scenario S5 undisturbed case.

	$G.\sigma$	$C.\gamma$	$Lev.\alpha$	$Lev.\gamma$	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.261	0.012	0.559	0.013	0.094	0.097	0.094	0.068	0.087	0.066
T_i	11.0	13.0	8.0	14.0	13.0	11.0	13.0	11.0	13.0	11.0
T_d	5.4	3.2	2.2	2.8	3.4	5.4	3.4	5.4	3.2	5.2
κ	8.6%	1.6%	34.5%	1.6%	1.9%	8.6%	1.9%	8.6%	1.6%	8.2%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

as Gauss, while IAE prefers performance close to Cauchy scaling.

Location detection robustness was done and basic statistics were calculated for PID scenario S5. As previously, only Cauchy location is robust to applied noises, although observation of average and median shows that almost all distributions except GEV and α -stable perform accurately.

Normal distribution is biased and is not decisive in loop variability assessment. Laplace PDF follows the same path. The following tables present detection results for undisturbed (Table 8) and disturbed cases (Table 9).

Analyzing contour plots, we observe that parameters of Cauchy distribution are still the only ones being robust. Only one disturbed case is presented (Figure 15). It is smooth. Normal standard deviation loses selection ability in case of disturbances. MSE is biased toward aggressive Gaussian solution, while IAE is in accordance with γ . There the following are concluded:

1. Disturbances significantly worsen proper assessment with shadowing effect.
2. Gaussian standard deviation σ is not appropriate.
3. Laplace is very similar to Gauss in disturbed case.
4. Scaling γ of Cauchy and Lévy is robust.
5. IAE performs better than MSE but is still shadowed.
6. GEV tends toward large gains and is ineffective.

Results presented above were obtained for a second order with delay process. Although it is quite representative for process industry, it is worth to extend the analysis for other processes.

Presentation of the obtained results is presented in a very similar way as for the previously analyzed PID controllers. For each of the plants, three scenarios are analyzed, evaluating performance indexes for the controller parameter surface, while one of the tuning parameters is frozen.

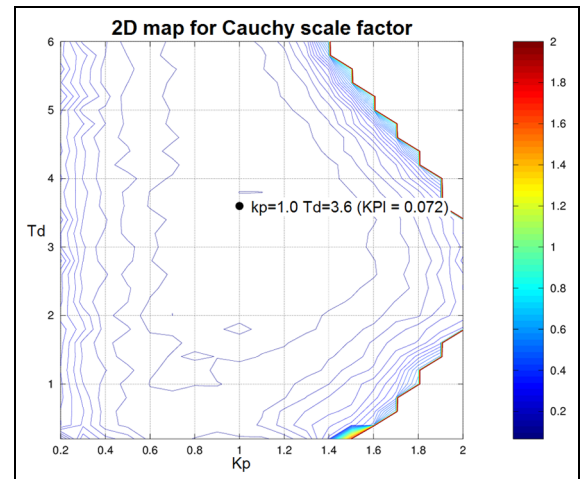
**Figure 14.** Detection for disturbed PID (S4) with Cauchy γ .

Table 9. PID controller scenario S5 disturbed case.

	G. σ	C. γ	Lev. α	Lev. γ	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.357	0.069	0.507	0.070	0.190	0.173	0.190	0.128	0.087	0.066
T_i	26.0	12.0	3.0	13.0	13.0	38.0	13.0	26.0	13.0	11.0
T_d	1.2	3.0	6.0	5.2	5.6	2.8	5.6	1.2	3.2	5.2
κ	0.0%	5.7%	85.3%	3.9%	4.8%	0.0%	4.8%	0.0%	1.6%	8.2%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

Table 10. PID controller for transfer function $G_1(s)$.

	G. σ	C. γ	Lev. α	Lev. γ	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.468	0.154	0.758	0.155	0.286	0.435	0.286	0.219	0.011	0.005
k_p	0.090	0.090	0.010	0.090	0.090	0.090	0.090	0.090	0.090	0.090
T_i	4.25	2.25	3.75	2.25	4.25	4.25	4.25	4.25	2.00	2.00
T_d	0.8662									
κ	1.99%	3.49%	5.46%	3.49%	1.99%	1.99%	1.99%	1.99%	4.32%	4.32%
min	0.874	0.170	0.761	0.176	0.411	1.448	0.411	0.764	0.012	0.006
k_p	0.0750	0.0900	0.0150	0.0850	0.0800	0.0800	0.0800	0.0750	0.0900	0.0900
T_i	2.4479									
T_d	0.55	0.65	0.60	0.45	0.65	0.45	0.65	0.55	0.50	0.90
κ	2.37%	2.14%	6.22%	2.01%	2.37%	2.13%	2.37%	2.37%	1.96%	2.60%
min	1.880	0.399	0.754	0.395	0.906	1.486	0.906	3.533	0.031	0.016
k_p	0.0172									
T_i	1.00	1.00	4.75	1.00	1.00	3.50	1.00	1.00	1.00	1.00
T_d	0.15	0.10	0.15	0.15	0.15	0.70	0.15	0.15	0.10	0.10
κ	11.44%	21.58%	3.90%	11.44%	11.44%	5.12%	11.44%	11.44%	21.58%	21.58%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

Thus, at first, T_d is fixed and two other settings vary. Next, the procedure is performed with frozen T_i and k_p . Tables show the best (minimum) value of each measure in each considered cross section. In addition, overshoot value for the detected tuning is added to enable comparison. The best solutions are highlighted with bold text.

Table 10 presents summary of results for transfer function $G_1(s)$ representing process with two dynamics, that is, fast and slow element. We see that controller performance does not vary much. Overshoot in all cases is quite similar despite differences in parameters. The shape of the performance surfaces is relatively flat. It is difficult to notice significant distinction between the solutions. Despite detection is difficult, possible risk of wrong tuning is minor.

Table 11 presents summary of the results obtained with transfer function with oscillatory properties. We see that scaling factor γ is very effective in detection and selects considerably better solutions than the other factors. We may also notice that stability parameter α is indecisive. In some cases, it shows very aggressive results, in the others sluggish ones. This behavior may be explained with two reasons: shadowing effect and the fact that the best performance is not associated with minimum of the stability parameter. One should expect that it should be close to 2.0. Smaller values may reflect persistent operation.

Table 12 shows results for a non-minimum phase plant. Data confirm previous observations that scaling factor γ , especially the one from Cauchy distribution can play the role of the effective descriptor of the control loop quality.

Industrial data

Simulations from the previous section enabled to formulate some hypotheses about assessment measures.

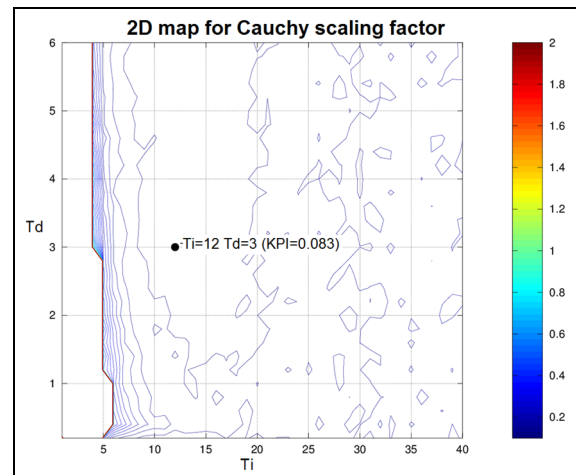
**Figure 15.** Detection for disturbed PID (S5) with Cauchy γ .

Table 11. PID controller for transfer function $G_2(s)$.

	G. σ	C. γ	Lev. α	Lev. γ	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.242	0.050	0.641	0.052	0.127	0.096	0.127	0.059	0.034	0.019
k_p	0.18	0.01	0.02	0.01	0.16	0.16	0.16	0.18	0.14	0.20
T_i	0.50	0.05	0.05	0.05	0.55	0.60	0.55	0.50	0.35	0.40
T_d	3.4884									
κ	19.88%	3.51%	0.00%	3.51%	14.84%	12.50%	14.84%	19.88%	12.14%	12.95%
min	0.245	0.052	0.122	0.054	0.128	0.086	0.128	0.06	0.034	0.020
k_p	0.14	0.08	0.09	0.08	0.14	0.14	0.14	0.14	0.14	0.2
T_i	0.2815									
T_d	4.00	3.75	0.50	3.75	4.00	4.25	4.00	4.00	3.75	4.25
κ	16.12%	3.58%	46.38%	3.58%	16.12%	13.93%	16.12%	16.12%	18.44%	30.79%
min	0.277	0.051	0.137	0.052	0.149	0.102	0.148	0.077	0.038	0.026
k_p	0.0797									
T_i	0.35	0.35	0.25	0.35	0.35	0.35	0.35	0.35	0.25	0.25
T_d	3.00	4.25	0.50	4.25	3.00	3.00	3.00	3.00	3.50	2.75
κ	0.83%	2.92%	16.39%	2.92%	0.83%	0.83%	0.83%	0.83%	4.17%	7.21%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

Table 12. PID controller for transfer function $G_3(s)$.

	G. σ	C. γ	Lev. α	Lev. γ	Lap.b	GEV.std	IAE	MSE	IAE_step	MSE_step
min	0.169	0.028	0.758	0.032	0.068	0.056	0.068	0.029	0.017	0.012
k_p	1.7	1.7	0.4	1.7	1.7	1.9	1.7	1.7	1.9	1.9
T_i	2.4	2.8	2.8	2.6	2.4	3.8	2.4	2.4	3.0	3.0
T_d	0.7339									
κ	12.67%	6.28%	0.00%	9.19%	12.67%	1.54%	12.67%	12.67%	7.97%	7.97%
min	0.161	0.027	0.73	0.028	0.061	0.054	0.061	0.260	0.016	0.011
k_p	1.80	1.70	0.30	1.80	1.80	1.80	1.80	1.80	2.00	2.00
T_i	2.4626									
T_d	1.6	1.2	0.4	1.3	1.2	1.6	1.2	1.6	1.1	1.3
κ	5.74%	2.83%	0.00%	3.50%	3.05%	5.74%	3.05%	5.74%	8.82%	8.64%
min	0.184	0.032	0.858	0.034	0.083	0.061	0.083	0.034	0.024	0.015
k_p	1.0556									
T_i	2.20	2.80	4.80	2.80	2.00	2.20	2.00	2.20	2.40	1.80
T_d	0.1	0.5	2	0.5	0.5	0.1	0.5	0.1	0.7	0.1
κ	6.48%	0.00%	0.00%	0.00%	13.11%	6.48%	13.11%	6.48%	1.32%	11.08%

Bold values describe the best solutions closest to the optimal solution. GEV: generalized extreme value; IAE: integral absolute error; MSE: mean square error.

Table 13. Assessment trial exemplary loops (B—before the tuning, A—after the tuning).

Loop	1		2		3		4		5	
	B	A	B	A	B	A	B	A	B	A
M/A stat	97.8%	100%	97.8%	100%	100%	100%	100%	100%	100%	100%
G.mean	0.33	−0.07	0.02	0.00	0.31	−0.38	0.01	−0.01	0.63	0.82
G.stdev	19.87	15.25	2.65	0.43	16.00	7.73	1.66	0.39	3.86	2.28
C.scale	12.24	9.33	1.42	0.24	6.60	4.41	0.42	0.13	2.15	0.75
Lev.stab	1.98	2.00	1.85	1.85	1.54	1.93	1.13	1.43	1.85	1.47
Lev.scale	13.67	10.78	1.68	0.27	7.26	5.17	0.44	0.14	2.46	0.81
Lap.b	15.75	12.16	2.01	0.33	10.47	5.97	0.90	0.22	2.99	1.31
Skewness	−0.04	0.08	0.26	0.43	0.09	0.10	−0.05	0.52	−0.13	3.86
Kurtosis	5.43	3.35	3.84	4.60	9.49	3.91	12.67	24.43	3.52	33.57
IAE	15.75	12.16	2.01	0.33	10.47	5.98	0.90	0.22	3.07	1.39
MSE	394.75	232.49	7.02	0.18	256.08	59.90	2.77	0.15	15.28	5.88

IAE: integral absolute error; MSE: mean square error.

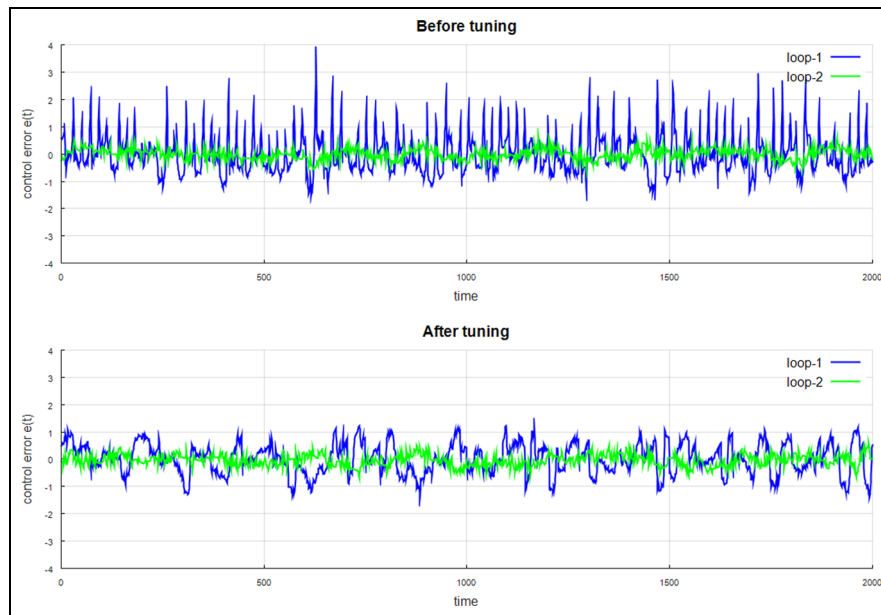


Figure 16. Exemplary control error trends for industrial loops.

The simulation environment was pretending industrial reality. But we should be aware that it is possible to only limited extent. Thus, exemplary validation of the approach with industrial data is presented. Data originate from anonymous (confidentiality subject) continuous-time gas processing plant. The main types of control as for pressure, temperature, flow and tank level can be found in the data. Control loops underwent general tuning initiative. The rehabilitation of the plant control system and its tuning was done by expert engineers. The work took several weeks. The validation of the whole initiative was done over the long period of several months of the process normal operation. Two data files are used: *before* and *after* tuning. There are 56 single element (SISO) PID (with parallel algorithm) loops. Control error signal is used as a variable for loop performance evaluation.

As there are many loops, it is difficult to show all data. Table 13 presents comparison of different measures calculated for five selected loops. It shows the values of measures for both situations *before* (denoted by **B**) and *after* (denoted by **A**) the tuning.

Exemplary trends for two selected variables for both situations: *before* and *after* the tuning are sketched in Figure 16.

Variables statistical properties

Data are at first tested for normality. Loops *before* reject normality hypothesis with Kolmogorov or Lilliefors tests, except one loop, for which Kolmogorov holds. Skewness or stability tests show normality in five cases. Fitting of α -stable PDF failed to deliver any results in case of two loops.

MSE for PDF fitting to histogram is evaluated. In majority of cases, α -stable PDF is the best fitted. It fits for 79% of loops, and Cauchy fits the best for 11% of

the loops. Gaussian distribution is best in fitting only in minority of cases (9%).

All *after* data reject normality hypothesis with Kolmogorov and Lilliefors. Skewness or stability tests depict normality in three cases. In majority, α -stable PDF is best fitted (75%), while 18% of them are best fitted with Cauchy. Gaussian distribution is applicable only in 13% of cases. GEV distribution succeeds for 9% and Laplace in 7% of the loops. Above confirms initial observation and assumption about non-Gaussian properties of the industrial variables.

Assessment trial assuming Gaussian hypothesis

As we have data reflecting loop performance before and after the tuning, we may check whether the measures can detect tuning improvement. Indexes for both data sets are compared. Gaussian measures detect improvement in 61% of cases. MSE shows better control in 64% of loops. IAE gives exactly the same results. However, if we focus on the loops with Gaussian properties, we will succeed with MSE in 86% of cases. IAE delivers good detection in 57% of cases. It should be noted that results, when Gaussian properties hold, are very good.

Assessment trial assuming non-Gaussian hypothesis

Non-Gaussian analysis focuses on factor γ of α -stable PDF. Parameters are compared for all the loops. γ shows good detection for 55% loops. Removal of all non-stable distributions shows effectiveness for σ and γ being equal to 65%. *After* loops that are non-Gaussian is good in 67% and 70% cases, respectively. Scaling γ is slightly better.

IAE gives bigger numbers than MSE, 68% versus 65% and 65% versus 62%, in both situations. There

are two reasons for that. Tuning of loops make error histograms closer to Gaussian. This effect is visible in share of better fitted PDFs. Fitting is very close to each other in many cases. It is expected that applicability will increase for poorly maintained loops or process being subject to larger disturbances.

Conclusion and open issues

This article presents results of the research on statistical CPA measures applied to assessment of control quality for PID SISO loops. Analysis starts with simulations. Real process data are used to validate observations. Assessment uses control error signal. Industrial data show that real signals often do not have Gaussian properties and feature long tails. It is caused by complex cross coupling, varying time delays, nonlinearities and human interventions. Cauchy disturbance is added in simulations to reflect real histograms shapes. Although detection works in undisturbed case, it rather fails in opposite situation. Applied disturbances shadow ability of proper assessment.

Standard integral measures of MSE and IAE are also compared. It is confirmed that MSE points out aggressive tuning. They both witness disturbance shadowing. We see that IAE is rather in-line with non-Gaussian PDFs, while MSE is close normal standard deviation. Concluding the observations are as follows:

1. Control quality assessment using statistical factors strongly depends on loop disturbance properties (shadowing effect). Control error should be first checked for statistical properties. Proper distribution function and its relevant parameters should be selected.
2. Gaussian approach is appropriate in case of no disturbances (low noise to signal ratio) or when loop properties are normal.
3. Analysis of control error histogram helps in selection of appropriate probabilistic density function. We may select it using distribution fitting to the histogram shape. It is suggested to use visual inspection of data.
4. Industrial data analysis shows that real data rarely have Gaussian properties. Fat-tail shape is more frequent. It causes shadowing effect that makes assessment difficult and unclear.
5. Scaling factor γ for Cauchy or Lévy α -stable probabilistic distribution is more appropriate choice and delivers better results as it is robust to shadowing effect.
6. GEV and Laplace functions rarely meet requirements.
7. Comparison of MSE and IAE shows that MSE selects aggressive tuning closer to detection with normal standard deviation. IAE is biased toward factors of long-tail functions and favors performance with low overshoots.

Above hypotheses confirm earlier observations done for other type of industrial control⁵ and simulation analysis performed for the GPC predictive control.²⁴

Automated and autonomous evaluation of indexes without reflection about loop environment may be misleading. Visual inspection of data, like trends, histograms, correlations and X-Y relations, is strongly recommended. Although such observation may be discouraging, it is reasonable. Thus, the following procedure is proposed:

1. Review time series of the variables. Verify if data are real, that is, without Supervisory Control and Data Acquisition (SCADA) compression effects and originating from normal process operating regimes.
2. Use control error signal as the assessment variable.
3. Draw control error histogram to evaluate variable statistical properties using the best fit probabilistic density function.
 1. In Gaussian case, normal standard deviation should be used as the loop quality measure.
 2. Once fat tails are detected, factors of Cauchy or α -stable distribution should be used. Scaling γ is the suggested choice, as it is the most robust against disturbances.
 3. Otherwise, select factors for selected specific function.
4. Choose appropriate measure protecting results biasing by disturbance shadowing.
5. MSE and IAE should be used with caution, although IAE seems to be a better solution.

Analysis leaves open issues to be addressed. The main question is associated with the shadowing effect, with its origins, impact strength and robust tools for its detection and measuring. Extensive research of industrial data is the must. Researches should work on investigation of relationship between real process phenomena and control error histogram properties. Clear and decisive indication would enable proper selection of assessment measures. Blindly selected distributions end up with random results.

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