# Multicriteria Subjective Reputation Management Model

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**Abstract.** The most widely used reputation models assume uniform users' preference structure. In this paper a new reputation management model is presented. It is focused on aggregation of community wide reputation in situation when agents do not share the same preference structure. The reputation is interpreted as vectors of attributes that represent several reputation evaluation criteria. Outcomes of the criteria are transformed by the utility functions and assigned subjective probabilities so that the subjective expected utility values can be obtained. Subjective expected utilities are further aggregated by the weighted ordered weighted average (WOWA) operator. The expressive power of subjective utilities concept along with the WOWA aggregation technique provides the reputation management system with a capability to model various preference structures. It is shown with an illustrative example.

## 1 Introduction

The problem of providing trust in virtual communities has drawn much attention ever since the widespread development of Web 2.0 applications. The purpose of trust and reputation systems is to strengthen the quality of markets and communities by providing an incentive for good quality services, and by sanctioning low quality services. One of key issues of any trust management system is how it deals with reputation. Reputation is defined as all available information about certain agent in a given community communicated by members of this community. An individual's subjective trust can be derived from a combination of received referrals and personal experience. There has been developed a number of reputation management models [7]. However, there are still valid questions that remain unanswered. Especially, if gathered reputation information can be left for an agent to process it according to his individual preferences. This is hard to accept, especially for human agents, due to the amount of reputation information (outcomes) to be analyzed. Therefore, many models ignore agents' subjective preference structures assuming that all participants of a network share the same view on available evidence and enforcing unified reputation measures.

This paper aims to present a solution to the question on how to automatically aggregate the reputation to support the decision of an agent while following its preference attitude. The proposed model allows to look at agents trust in the

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point of multiple evaluation criteria. It maintains the original multiattribute reputation outcome distributions for evaluation criteria and enables the trusting agents to express their preference on the outcomes when estimating trust in other agents. The model makes it possible for the agents to put importance weights on the evaluation criteria. It combines the Weighted Ordered Weighted Average (WOWA) operator with the concepts of subjective expected utility theory to build the model that can be adjusted to the users' preferential structures.

The idea of application of fuzzy aggregation operators to the problem of reputation management has appeared in various works. For example, the Ordered Weighted Average (OWA) [1] or the WOWA [2] usage were analyzed within this context. However, both these approaches do not allow to take into account individual agent preferences as the fuzzy aggregation operators are applied to unified scalar reputation scores. Lee *et al.* [8] developed a fuzzy trust model which takes into account both evaluations from multiple criteria and the recommendations from others in order to set the trust degrees on entities. In the model, the entity's preference degrees on the outcomes of the interactions are expressed in fuzzy sets and the trust degrees are determined by aggregating the satisfaction degrees with respect to evaluation criteria with Sugeno fuzzy integral. The WOWA aggregation used in our model provides, however, much simpler and more transparent agents preference modeling. Moreover, it opens a possibility to incorporate into the reputation analysis the multicriteria decision support techniques such as the Reference Point Method (RPM) [17]. The RPM interactive analysis is navigated with the commonly accepted control parameters expressing reference levels for the individual criteria and it can be based on the WOWA aggregation of appropriate achievement measures [9,10].

## 2 Subjective Probability

There exist a number of probability interpretations [6], however the notion of probability has basically dual understanding. The first concept of probability is based on the observation of relative frequency of outcomes in the repeated experiments. As the number of experiments increases the parameters of such empirical distribution approach the real "objective" probability distribution of the outcomes. Apart from the above "Bernoulli type" of probability the other, equally old interpretation of probability states that the probability reflects beliefs that certain outcome will obtain. This view of probability lead Ramsey [12] to the concept of "subjective probability", that was further formalized by de Finetti [3]. The general assumption that allows to use agents' subjective probabilities is that they follow probability calculus rules.

### 2.1 Savage's Subjective Expected Utility Theory

Savage [14] has proposed framework to deal with subjective probabilities and utilities. Savage's framework consists of the following elements:

- *states of the world* as possible scenarios of the future with only one *true state*;
- *consequences* entities, that have value to the decision maker;
- *acts* functions that associate consequences with states;
- *events* the subsets of state space.

Moreover, Savage has developed a set of 7 postulates that define the preference structure:

1. The preference relation between acts is complete (each two acts are comparable) and transitive.
2. The decision between acts is based only on the consequences in the states when the consequences are distinct.
3. The ordering of consequences is state and act independent.
4. The decision maker assigns probabilities to events with no regard to the consequences. Other words, the subjective probability of an event will not change even if the payoff's will change (preserving the ordering).
5. There exists at least one act that is preferred to some other act.
6. The state space is continuous. It is always possible to divide an event into smaller sub events and adjust probabilities accordingly.
7. The act "better", on each of the states of a certain event, then the other act is strictly preferred.

The preference relation is defined by the first four axioms, last three play rather technical role. Savage claims that the preference relation described by above seven postulates is analogous to the problem of expected utility maximization, when the utility function is defined on the set of consequences and the (subjective) probability measure is defined on the set of all events.

## 3 Multicriteria Trust Model Definition

Proposed reputation management model is based on the assumption of existence of subjective expected utility and subjective probabilities especially. The process of calculating reputation metric can be viewed as a process of calculating utilities on two levels. The first level of computations is done with respect to a single criterion. This involves deriving and applying utility function on the set of possible outcomes with subjective probabilities based on the reputation. The above leads us to the formulation of expected utility of the outcomes as a satisfaction level of the given criteria. The second level is the decision problem of selecting the best option (most trustworthy) among a number of offers, each described by a set of evaluation criteria. One can employ a variety of methods of multidimensional analysis to solve this problem. Interactive methods, like the reference point methods, can be used as well as the expected utility maximization approach can be applied.

### 3.1 Satisfaction Levels

Let us assume we have a set $C$ of $n$ possible criteria. For each criterion $c_i$ ($i \in C$) there is a set of possible outcomes $O^i$. Each of $o \in O^i$ has a subjective probability assigned that reflects users belief on certain value to occur in the next interaction. There is a preference relation $\succeq$ on set $O^i$ that follows usual assumptions of transitivity and completeness. The expression $o_1 \succ o_2$ means that the $o_1$ is more desirable then $o_2$ and $o_1 \sim o_2$ means that $o_1$ is equally desirable as $o_2$. If we consider the above model with respect to the language of Savage's framework then the state space can be regarded as a space of all possible transaction results. Each possible result of the transaction can be assigned one of the values from the set of consequences $O^i$. The assignment is done by the selection of a given option (act). Each option is comparable with the others. The ranking of options depends only on the ranking of possible transactions results where either the probability or the outcome are different. The ranking of outcomes is defined above and is independent of the transactions that yield them. The probabilities are calculated based on the reputation reports. The above defined preference structure on acts follow Savage's axioms of rationality of preference structure thus modeling it with the subjective expected utility is justified.

Set $R^i$ is the set of evaluations (reputation) that refer to the criterion $c_i$ Set $R^i_o$ is a set of evaluations with outcome $o$ of the criterion $c_i$ The subjective probability of an outcome $o$ corresponds to the available reputation and is defined as:

$$sp(o) = \frac{|R^i_o|}{|R^i|}$$

This is not the objective probability measure as the set of reputation valuations is not a set of independent repetitive trials of some phenomenon. Instead it reflects user's attitude towards the possible outcome of the transaction, since in the reputation system past interactions and reputation are the only source of knowledge that influence user's beliefs. Probabilities $sp(o)$ are calculated for all outcomes. If a given outcome has never occurred then the probability is assumed as equal to 0.

The degree of preference relation between outcomes has to be incorporated into the preference structure also. In order to reasonably model user's preferences we need to transform them into a common measurement scale that is going to reflect the preferential order of outcomes as well as to measure the strength of the relation. Let the measurement (utility) function be denoted by $u$. Utilities are normalized to sum up to one. The utility function needs to be consistent with the preference relation $\succeq$ on set the of outcomes $O$. The Subjective Expected Utility of a particular criterion $c_i$ is finally calculated as

$$x_i = \frac{1}{|R^i|} \sum_{o \in O^i} |R^i_o| u(o) \qquad (1)$$

### 3.2 Reputation Score

In the reputation management model presented in this paper the reputation scores are generated by aggregating subjective expected utilities of the criteria using the WOWA operator. The standard *Ordered Weighted Average* (OWA) operator [18] allows one to introduce preferential weights assigned to the ordered values of the aggregated vector elements rather than particular elements of the vector. Formal definition of the operator is as follows. Given vector of $n$ values $x_i$ for $i = 1, \ldots, n$ and preferential weights vector $w_i \geq 0$ for $i = 1, \ldots, n$ while $\sum_{i=1}^{n} w_i = 1$. The OWA operator is defined as:

$$\sum_{i=1}^{n} w_i x_{\sigma(i)}$$

where $\sigma(i)$ is a permutation ordering vector $x$ from the largest to the smallest element:

$$x_{\sigma(1)} \geq x_{\sigma(2)} \geq x_{\sigma(3)} \geq \ldots \geq x_{\sigma(n)}.$$

The OWA operator allows one to model various aggregation functions from the maximum through the arithmetic mean to the minimum. Thus, it enables modeling various preferences from the optimistic to the pessimistic one. On the other hand, the OWA does not allow one to allocate any importance weights to specific criteria. Several attempts have been made to incorporate importance weighting into the OWA operator. Finally, Torra [15] has incorporated importance weighting into the OWA operator within the Weighted OWA (WOWA) aggregation. The WOWA averaging is based on two weighting vectors:

- preferential weights vector $\boldsymbol{w}$ ($w_i \geq 0$, $\sum_i w_i = 1$) associated with criteria satisfaction level values ordered from the highest value to the lowest.
- importance weights vector $\boldsymbol{p}$ ($p_i \geq 0$, $\sum_i p_i = 1$) associated with the aggregated criteria.

Actually, the WOWA average is a particular case of Choquet integral using a distorted probability as the measure [16].

Formal WOWA definition follows the OWA formula aggregating the values ordered from the highest to the lowest one:

$$WOWA(x_1, \ldots, x_n) = \sum_{i=1}^{n} \omega_i x_{\sigma(i)} \tag{2}$$

while weights $\omega$ are constructed by cumulation of the preferential weights $w_{\sigma(i)}$ and their decumulation according to the corresponding distribution of importance weights $p_{\sigma(i)}$, i.e.,

$$\omega_i = w^*(\sum_{j \leq i} p_{\sigma(j)}) - w^*(\sum_{j < i} p_{\sigma(j)}) \tag{3}$$

where function $w^*$ interpolates points $(i/n, \sum_{j \leq i} w_j)$ and point $(0, 0)$. When preferential weights $p_i$ are equal, WOWA becomes the standard OWA operator with preferential weights $w_i$. When preferential weights are equal, the WOWA operator becomes the weighted average operator. The WOWA aggregation generalizes both the OWA and the weighted average.

Alternatively, the WOWA aggregation may be given by the formula [11]:

$$WOWA(x) = \sum_{k=1}^{n} w_k n \int_{(k-1)/n}^{k/n} \overline{F}_x^{(-1)}(\xi)\ d\xi \tag{4}$$

where $\overline{F}_x^{(-1)}$ is the stepwise function $\overline{F}_x^{(-1)}(\xi) = x_{\sigma(i)}$ for $\alpha_{i-1} < \xi \leq \alpha_i$ with breakpoints $\alpha_i = \sum_{k \leq i} p_{\tau(k)}$ and $\alpha_0 = 0$. It can also be mathematically formalized as the left-continuous inverse $\overline{F}_x^{(-1)}(\xi) = F_x^{(-1)}(1-\xi)$ of the cumulative distribution function

$$F_x(d) = \sum_{i=1}^{n} p_i \delta_i(d) \quad \text{where} \quad \delta_i(d) = \begin{cases} 1 & \text{if } x_i \leq d \\ 0 & \text{otherwise} \end{cases}$$

Note that $n \int_{(k-1)/n}^{k/n} \overline{F}_x^{(-1)}(\xi)\ d\xi$ represents the average within the $k$-th portion of $1/n$ largest outcomes, the corresponding conditional mean. Hence, formula (4) defines WOWA aggregations with preferential weights $\boldsymbol{w}$ as the corresponding OWA aggregation but applied to the conditional means calculated according to the importance weights $\boldsymbol{p}$ instead of the original outcomes.

In case of the reputation management model presented in this paper subjective expected utilities of the criteria are aggregated using the WOWA operator. Having decided about the weighting vectors, following the WOWA formula (2)–(3), weights $\omega$ are calculated and the final value of the WOWA aggregation is derived. The calculated reputation score of each alternative agent is used to rank them in decreasing order. The agent with the highest score should be the most trusted one according to the user's preference structure.[1]

The definition of weights $\boldsymbol{w}$ induces certain shape of function $w^*$ and thereby allows us to model the user's attitude towards given decision situation [11]. Increasing weights $w_i$ (convex function $w^*$) shows user's diffident approach as it amplifies the impact of low values while reducing the importance of higher values. It requires all satisfaction levels be high enough to yield high aggregation value. On the other hand, decreasing sequence of weights $w_i$ (concave function $w^*$) is bound to the confident attitude since it amplifies higher values. It allows any of the criteria be highly satisfied to yield a high aggregated value. The whole range of $w^*$ shapes can be interpreted as a variety of users preference structures what makes WOWA aggregation so well suited for this sort of applications.

## 4 Illustrative Example

Auction service data usually allows user to leave some kind of rating for the other party of the transaction. Different rules may govern this process. Sometimes only

[1] In general case the actual decision should not rely only on reputation. For example, one should not neglect the impact of agents own experience with evaluated partners.

buyer is entitled to leave comment, sometimes party's comment is not revealed until other agent leaves his own evaluation of the transaction. Ratings are usually limited to the set of positive, neutral or negative evaluation accompanied by the free hand opinion on what actually happened. Auction house *eBay Inc.* has extended feedback system introducing a wider seller rating system. This functionality allows buyers to asses sellers across four dimensions assigning values of one to five to each of the following criteria [4]:

1. How accurate was the item description?
2. How satisfied were you with the seller's communication?
3. How quickly did the seller ship the item?
4. How reasonable were the shipping and handling charges?

Let us further assume that the user (bidder) has to choose the most trustworthy partner among 5 alternative auction users (sellers) who participated in a different numbers of transactions. Namely:

– Alice - Very good performance at describing items but fails at communication while maintaining average performance at shipment and handling charges. Very heavy user took part in the greatest number of transactions
– Bob - Average performer at all criteria. Average user with regard to the number of transactions performed.
– Carl - Outstanding communication and description but poor at any other criteria. Average transactions number.
– David - No handling charges and quick shipment while poor description and average communication skills. Average transactions number.
– Edward - Average at all dimensions but very low number of transactions performed.

Let us assume that we have identified 4 criteria that govern the decision process of auction house user and we are able to determine outcome domain of each criterion.

*Description* how accurate was the item description?

1. poor ($o_1^d$) $u(o_1^d) = 1$
2. good ($o_2^d$) $u(o_2^d) = 5$

| $c_1$ | | |
|---|---|---|
| | $o_1^d$ | $o_2^d$ |
| Alice | 1 | 100 |
| Bob | 25 | 25 |
| Carl | 2 | 28 |
| David | 28 | 3 |
| Edward | 2 | 2 |

*Communication* how satisfied were you with the seller's communication?

1. poor ($o_1^c$) $u(o_1^c) = 1$
2. medium($o_2^c$) $u(o_2^c) = 2$
3. good($o_3^c$) $u(o_3^c) = 3$
4. very good ($o_4^c$) $u(o_4^c) = 4$
5. outstanding ($o_5^c$) $u(o_5^c) = 5$

| $c_2$ | | | | | |
|---|---|---|---|---|---|
| | $o_1^c$ | $o_2^c$ | $o_3^c$ | $o_4^c$ | $o_5^c$ |
| Alice | 40 | 20 | 15 | 2 | 1 |
| Bob | 5 | 8 | 8 | 7 | 7 |
| Carl | 0 | 0 | 0 | 20 | 10 |
| David | 6 | 14 | 9 | 1 | 0 |
| Edward | 1 | 2 | 1 | 1 | 1 |

*Shipment* how much time took the delivery?

1. poor ($o_1^s$) $u(o_1^s) = 1$
2. medium($o_2^s$) $u(o_2^s) = 2$
3. good ($o_3^s$) $u(o_3^s) = 3$
4. very good ($o_4^s$) $u(o_4^s) = 4$

| $c_3$ | | | | |
|---|---|---|---|---|
| | $o_1^s$ | $o_2^s$ | $o_3^s$ | $o_4^s$ |
| Alice | 2 | 40 | 40 | 2 |
| Bob | 4 | 12 | 12 | 8 |
| Carl | 19 | 6 | 1 | 0 |
| David | 0 | 0 | 10 | 20 |
| Edward | 0 | 2 | 2 | 1 |

*Handling* how reasonable were the shipping and handling charges?

1. poor ($o_1^h$) $u(o_1^h) = 1$
2. medium ($o_2^h$) $u(o_2^h) = 2$
3. good ($o_3^h$) $u(o_3^h) = 3$
4. very good ($o_4^h$) $u(o_4^h) = 4$
5. outstanding ($o_5^h$) $u(o_5^h) = 5$

| $c_4$ | | | | | |
|---|---|---|---|---|---|
| | $o_1^h$ | $o_2^h$ | $o_3^h$ | $o_4^h$ | $o_5^h$ |
| Alice | 5 | 30 | 40 | 30 | 5 |
| Bob | 1 | 12 | 18 | 13 | 1 |
| Carl | 24 | 9 | 2 | 0 | 0 |
| David | 0 | 0 | 0 | 0 | 30 |
| Edward | 1 | 3 | 3 | 1 | 0 |

User specifies his preferences first by assigning utilities to the outcomes as it was described above. The selection of utilities leads to the following subjective expected utilities:

| criterion: | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| Alice | 0.827 | 0.118 | 0.229 | 0.200 |
| Bob | 0.500 | 0.206 | 0.263 | 0.201 |
| Carl | 0.789 | 0.289 | 0.119 | 0.091 |
| David | 0.231 | 0.144 | 0.394 | 0.333 |
| Edward | 0.500 | 0.189 | 0.273 | 0.167 |

Consider various results based on the WOWA aggregation defined with by several selections of the vector of preferential weights and importance weights.

*OWA aggregation case.* We suppose that user has no special preferences with regard to the criteria in question. All of them are equally weighted. Since we have only 4 criteria each element of the $\boldsymbol{p}$ is equal, so $\boldsymbol{p} = [0.25, 0.25, 0.25, 0.25]$. User though has the preferences regarding achievements of the criteria. First, we shall assume user has expressed diffident approach and requires small values to contribute more to the result of aggregation. To achieve this the $\boldsymbol{w}$ vector should be selected in a way that leads to the convex interpolation function $w^*$. In the case considered $\boldsymbol{w} = [0.03, 0.05, 0.28, 0.65]$.

*Weighted mean.* We consider case when the reputation system user expresses his indifference with respect to the satisfaction values of the criteria providing vector $\boldsymbol{w} = [0.25, 0.25, 0.25, 0.25]$ and strong preference for the "description conformity" criterion giving vector $\boldsymbol{p} = [0.67, 0.09, 0.04, 0.20]$.

*General case.* If we now consider the most general case we shall express both preferences with respect to the criteria as well as for the achievement levels. Suppose the user expressed diffident approach but with the strong preference for the description component. Let us further assume he had provided following values for the weighting vectors: $\boldsymbol{p} = [0.63, 0.19, 0.06, 0.13]$ and $\boldsymbol{w} = [0.06, 0.13, 0.19, 0.63]$. Results of all three experiments are presented in the table below

| | Alice | Bob | Carl | David | Edward |
|---|---|---|---|---|---|
| owa | 0.164 | **0.213** | 0.127 | 0.184 | 0,013 |
| wm | **0.611** | 0.403 | 0.593 | 0.25 | 0,051 |
| general | 0.339 | 0.29 | **0.349** | 0.196 | 0.017 |

The first case would lead to the selection of *Bob* while the weighted mean aggregation would prefer *Alice*. The general WOWA aggregation case shows *Carl* as the best choice.

## 5 Concluding Remarks

The role of the information system based on the presented approach is to retrieve and process data according to the user's preference structure and to provide result as a single scalar rating. What is probably the most important users get instant knowledge on trustfulness of the possible transaction partners while the effort of retrieving and analyzing feedback is done by the system. When there is no extended seller's ratings much can be achieved by analyzing feedback text to retrieve opinions on the selected criteria space. Recent developments in text mining and natural language processing can be exploited in this area. Specifying both utilities of certain outcomes of the criteria as well as two weight vectors ($\boldsymbol{p}$ and $\boldsymbol{w}$) may be confusing for the average user. There is still a lot of work to do on how to retrieve user's individual preferences and transform them into utilities and WOWA weighting vectors [5,16].

The preferential weights definition can be simplified by allowing to introduce scalable preferences with weights allocated to specific portion of the worst outcomes independently from the number of criteria. Formula (4) allows us to define such a generalized WOWA aggregation [11] where the preferential weights $w_k$ are allocated to an arbitrarily defined grid of ordered outcomes defined by $m$ (independent of $n$) quantile breakpoints $\beta_0 = 0 < \beta_1 < \ldots < \beta_{m-1} < \beta_m = 1$, i.e. the aggregation defined with a piecewise linear function $w^*_\beta$ interpolating points $(\beta_k, \sum_{i\leq k} w_i)$ together with the point (0.0). Moreover, the WOWA aggregation opens a possibility to incorporate into the reputation analysis the multicriteria decision support techniques such as the Reference Point Method (RPM) [17]. The RPM interactive analysis is navigated with the commonly accepted control parameters expressing reference levels for the individual criteria and it can be based on the WOWA aggregation of appropriate achievement measures [9,10].

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## References


1. Aringhieri, R., Damiani, E., De Capitani Di Vimercati, S., Paraboschi, S., Samarati, P.: Fuzzy techniques for trust and reputation management in anonymous peer-to-peer systems. Journal of the American Society for Information Science and Technology 57, 528–537 (2006)
2. Damiani, E., De Capitani Vimercati, S., Samarati, P., Viviani, M.: A WOWA-based Aggregation Technique on Trust Values Connected to Metadata. Electronic Notes in Theoretical Computer Science 157, 131–142 (2006)
3. De Finetti, B.: Foresight: its logical laws in subjective sources. In: Studies in Subjective Probability, pp. 93–158 (1964)
4. Ebay Inc.: `http://www.ebay.com`
5. Filev, D., Yager, R.R.: On the issue of obtaining OWA operator weights. Fuzzy Sets and Systems 94, 157–169 (1998)
6. Hajek, A.: Interpretations of Probability. The Stanford Encyclopedia of Philosophy (2009)
7. Josang, A., Ismail, R., Boyd, C.: A Survey of Trust and Reputation Systems for Online Service Provision. Decision Support Systems 43, 618–644 (2007)
8. Lee, K., Hwang, K., Lee, J., Kim, H.J.: A Fuzzy Trust Model Using Multiple Evaluation Criteria. In: Wang, L., Jiao, L., Shi, G., Li, X., Liu, J. (eds.) FSKD 2006. LNCS (LNAI), vol. 4223, pp. 961–969. Springer, Heidelberg (2006)
9. Ogryczak, W.: Ordered weighted enhancement of preference modeling in the reference point method for multiple criteria optimization. Soft Computing 14, 435–450 (2010)
10. Ogryczak, W., Kozłowski, B.: Reference point method with importance weighted ordered partial achievements. TOP (2009), Springer Online First doi:10.1007/s11750-009-0121-4
11. Ogryczak, W., Śliwiński, T.: On Efficient WOWA Optimization for Decision Support under Risk. International Journal of Approximate Reasoning 50, 915–928 (2009)
12. Ramsey, F.P.: Truth and probability. The Foundations of Mathematics and other Logical Essays 7, 156–198 (1926)
13. Resnick, P., Zeckhauser, R., Friedman, E., Kuwabara, K.: Reputation Systems. Communications of the ACM 43, 45–48 (2000)
14. Savage, L.J.: The Foundations of Statistics. Wiley, New York (1954)
15. Torra, V.: The Weighted OWA Operator. International Journal of Inteligent Systems 12, 153–166 (1997)
16. Torra, V., Narukawa, Y.: Modeling Decisions: Information Fusion and Aggregation Operators. Springer, Berlin (2007)
17. Wierzbicki, A.P.: Reference point approaches. In: Gal, T., Stewart, T., Hanne, T. (eds.) Multicriteria Decision Making: Advances in MCDM Models, Algorithms, Theory, and Applications, pp. 9.1–9.39. Kluwer, Dordrecht (1999)
18. Yager, R.R.: On ordered weighted averaging aggregation operators in multicriteria decisionmaking. IEEE Transactions on Systems, Man and Cybernetics 18, 183–190 (1988)