PERGAMON Computers and Mathematics with Applications 37 (1999) 135–150

An International Journal
computers & mathematics
with applications

Relocation Problems Arising in Conservation Biology

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Abstract—An early motivation for this study was the problem of relocation of scarce or endangered species of animals for breeding and/or reintroduction to establish new populations in the wild. In this paper, we introduce single and multiple objective optimization models which are designed to comprehend a wide variety of objectives which are of interest to conservation and wildlife managers. We present the models in a general way and point out special features relative to ecology as they arise. Thus, the models may be used for relocation decisions analysis in diverse fields, not only in conservation and ecology. After presentation of the models in such a general way, we reformulate the models to make use of the special structure present. Such reformulation reduces the number of decision variables and constraints and, in general, makes solutions easy to obtain. By easy to obtain, we mean that tools from linear and mixed-integer programming together with elementary sorting procedures provide the basis for solving the models.

In order to illustrate the capabilities of the models and solution techniques developed, we present the results of their application to the real-life relocation problem arising while analyzing restoration of the globally endangered Przewalski's horse population.

Keywords—Multiple objectives, Wildlife management of endangered species.

1. INTRODUCTION

An early motivation for this study was the problem of relocating individuals from rare or endangered species into breeding or reintroduction programs in such a way as to preserve the greatest amount of genetic diversity. Managers of captive breeding programs for literally dozens of endangered species attempt to optimize these selections each year, but they are often frustrated by the astronomical number of choices. We show here that the genetic optimization problems can be treated naturally as single and multiple objective integer programming problems, though with discrete decision variables and nonlinear objective functions. These are not necessarily solvable with real or realistic data. However, we develop some transformations which allow these problems to be solved, and illustrate these techniques with actual data from the breeding program for the globally endangered Przewalski's or Asiatic wild horse [1].

The authors would like to thank D. Bannister, C. Cone, and J. Shaw who made contributions through some projects in their degree work in the Biological and Mathematical Sciences Departments of Clemson University. We also wish to thank O. Ryder for sharing data on Przewalski's horse, and Zoo Atlanta and the Harriet Jackson Phelps Charitable Trust for their financial support.

PII: S0898-1221(99)00065-6

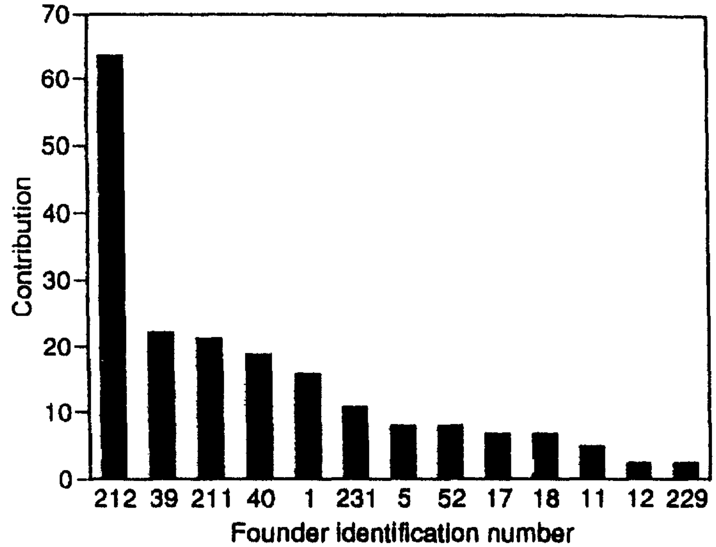
This horse is the only true wild species of horse on earth, distinct from feral domestic horses, though it is extinct 'in the wild'. The current world population of over 1200 animals is descended entirely from just 13 individuals, termed founders. Since each individual receives half of its genes from each parent, and we know the pedigree of the entire world population, we can calculate the fraction of each individual's genes ultimately derived from each of the 13 founders. Summing over all 191 animals in the North American captive population, we see that the result of about a dozen generations of captive breeding has led to the favoring of some founders' genes at the expense of others, and a loss of perhaps two thirds of the total genetic variation in the founders (see Figure 1). However, it is now believed that the continued loss of genetic variability can threaten the survival of the animals, and managers of this and many other endangered species attempt to choose individuals for breeding and reintroduction that best preserve what genetic variation remains. Unfortunately, even with complete information, as with the Asiatic wild horse, managers can be frustrated in achieving their goals by the huge number of choices possible. This is where our approach may be of help.

After studying several models, we realized that the situation described above is one of more general interest. There are needs for relocation in diverse fields, not only in conservation and ecology. Thus, we wish to present the models in a general way and point out special features relative to ecology as they arise. After presentation of the models in such a general way, we reformulate the models to make use of the special structure present. Such reformulation reduces the number of decision variables and the number of constraints, and, in general, makes solutions easy to obtain. By easy to obtain, we mean that tools from linear and mixed-integer programming together with elementary sorting procedures provide the basis for solving the models. Such technology is currently available, and hence, we obtain a set of models which is quite usable for the general audience of operations researchers, together with computer scientists, engineers, and business analysts. Included in the paper are details about the solution of the models, including proofs of the validity of simple sorting on certain models, and discussion about a more complex form of sorting which is applicable to one model in the reformulated setting. It is felt that the use of sorting to solve the complex decision models is quite remarkable in terms of potential impact for solving large models and in terms of providing mathematical justification for what is one of the heuristics which has been used in the management of endangered species. With linear and integer programming software easily available, and with advances in computer technology, it is felt that solutions for all reasonably sized problems are obtainable. Hence, we suggest that the value of making such mathematical models available will be realized by this paper, while new solution methods are not necessary for proper application of the models.

The paper is organized as follows. In Section 2, we start with initial optimization models depicting the decision problem in a way that all relationships to the data and to the logical framework of decision making are expressed explicitly. In Section 3, we reformulate the models to make use of the special structure present. Such reformulation reduces the number of decision variables and the number of constraints. Next, in Section 4, we discuss the solution techniques for the optimization models. It turns out that most of them can be solved with simple sorting procedures. Finally, in Section 5, the case study of the Przewalski's horse herd restoration is presented.

2. BASIC MODELS

In this section, the formulations of the mathematical models in their first version (or primary version) are developed. Here all relationships to the data and to the logical framework of decision making are expressed explicitly. These models are the most natural way of thinking about the problem, but they are also the largest in terms of the number of decision variables and constraints and storage requirements. To think that all of them could be solved routinely would be naive. However, it is worthwhile to keep all features of the models open for some time since it is likely



		Founder Identification Number												
		212	39	211	40	1	231	5	52	17	18	11	12	229
Descendant ID														
1	380	0.422	0.086	0.141	0.07	0.125	0	0.063	0.063	0.016	0.016	0	0	0
2	406	0.375	0.172	0.125	0.141	0.063	0	0.031	0.031	0.031	0.031	0	0	0
3	458	0.422	0.086	0.141	0.07	0.125	0	0.063	0.063	0.016	0.016	0	0	0
	⋮													
95	1515	0.258	0.104	0.086	0.092	0.078	0.031	0.039	0.039	0.051	0.051	0.086	0.043	0.043
96	1536	0.375	0.086	0.125	0.07	0.156	0	0.078	0.078	0.016	0.016	0	0	0
97	1539	0.375	0.043	0.125	0.035	0.203	0	0.102	0.102	0.008	0.008	0	0	0
	⋮													
189	5054	0.352	0.132	0.117	0.111	0.055	0.109	0.028	0.028	0.035	0.035	0	0	0
190	5055	0.346	0.129	0.115	0.11	0.055	0.109	0.028	0.028	0.041	0.041	0	0	0
191	5056	0.281	0.031	0.094	0.031	0.125	0.25	0.063	0.063	0.031	0.031	0	0	0
Total Contributions		63.61	22.11	21.21	18.7	15.72	10.67	7.868	7.868	6.763	6.763	4.93	2.468	2.468
Fractional Contributions		0.333	0.116	0.111	0.098	0.082	0.056	0.041	0.041	0.035	0.035	0.026	0.013	0.013
Number of Descendants		190	191	190	191	187	95	187	187	190	190	63	63	63

Figure 1. Founder contributions to all 191 Przewalski's horses in the North American SSP, as of 12-31-1992.

that there will be interest in developing other related models which may not simplify in the same way under reformulation. We are confident that there are many related problems outside conservation biology which take the same form as those developed here, but we will consistently relate back to the founder-descendant relocation problem to clarify the explanation.

Now, we introduce the terminology of the models. Let us define

d_j (an integer) demand for carriers at destination j, $j = 1, 2, \ldots, n$,

s_i (an integer) supply of carriers at location i, $i = 1, 2, \ldots, m$,

x_{ij} number of carriers transported from location i to destination j, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

Constraints on x_{ij} variables take the form of a typical transportation problem with m supply points (locations) and n destinations.

$$\sum_{j=1}^{n} x_{ij} = s_i, \qquad \text{for } i = 1,2,\dots,m, \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} = d_j, \qquad \text{for } j = 1,2,\dots,n, \tag{2}$$

$$x_{ij} \geq 0,\ x_{ij} \text{ integer} \qquad \text{for } i = 1,2,\dots,m,\ j = 1,2,\dots,n. \tag{3}$$

Each carrier transports some amounts of several properties. We consider r different properties. Let p_{ik} denote the amount of property k related to each carrier in i, $i = 1,2,\dots,m$, $k = 1,2,\dots,r$. We are interested in maximization of amounts of properties transported with carriers relocated to destinations $1,2,\dots,n-1$ (i.e., to all the destinations apart from the destination n). The objective function may be formulated in various ways.

Let us maximize the minimum over all properties ($k = 1,2,\dots,r$) of the total of the k^{th} property transported to one of destinations $1,2,\dots,n-1$. For algebraic formulation of this objective, we introduce auxiliary integer variables

y_{ik} number of units of property k transported from location i to one of destinations $1,2,\dots,$ $n-1$, ($i = 1,2,\dots,m$, $k = 1,2,\dots,r$).

Auxiliary variables y_{ik} are related to the decision variables x_{ij} by the following formulae

$$y_{ik} = \sum_{j=1}^{n-1} x_{ij}, \qquad \text{for } i = 1,2,\dots,m,\ k = 1,2,\dots,r. \tag{4}$$

The optimization problem can be stated then, as follows

$$\text{SOSUM:} \qquad \max_{X,Y} \left\{ \min_{k=1,\dots,r} \left\{ \sum_{i=1}^{m} p_{ik} y_{ik} : \text{ s.t. (1)–(4)} \right\} \right\}. \tag{5}$$

As another objective one may consider maximization of the minimum over all properties ($k = 1,2,\dots,r$) of the minimal amount of the k^{th} property transported to one of destinations $1,2,\dots,$ $n-1$ by a single carrier. For an algebraic formulation of this objective, we introduce auxiliary binary variables $u_{ik} \in \{0,1\}$, ($i = 1,2,\dots,m$, $k = 1,2,\dots,r$). Let $u_{ik} = 1$ indicate that at least one unit of property k is transported from location i to one of destinations $1,2\dots,n-1$, and $u_{ik} = 0$ otherwise. Then, auxiliary variables u_{ik} are related to the decision variables x_{ij} by the following

$$u_{ik} = 1, \qquad \text{iff } \sum_{j=1}^{n-1} x_{ij} > 0.$$

These relations can be expressed in the algebraic form with the following inequalities:

$$s_i u_{ik} \geq \sum_{j=1}^{n-1} x_{ij} \geq u_{ik}, \qquad \text{for } i = 1,2,\dots,m,\ k = 1,2,\dots,r. \tag{6}$$

The optimization problem can be stated then as follows:

$$\text{SOBOT:} \qquad \max_{X,U} \left\{ \min_{k=1,\dots,r} \left\{ \min_{i=1,\dots,m} p_{ik} u_{ik} : \text{ s.t. (1)–(3), (6) and } u_{ik} = 1 \right\} \right\}. \tag{7}$$

Transportation of several properties may be regarded as independent criteria. This leads us to the multiple objective formulations of the optimization problem with r criteria corresponding to several properties.

Model SOSUM can be generalized to a multiple objective optimization problem with r objective functions

$$\text{MOSUM:} \qquad \max_{X,Y} \left\{ \left\{ \sum_{i=1}^{m} p_{ik} y_{ik} \right\}_{k=1,\dots,r} : \text{ s.t. (1)–(4)} \right\}. \tag{8}$$

Similarly, model SOBOT can be generalized to the following multiple objective optimization problem with r objective functions

$$\text{MOBOT:} \qquad \max_{X,U} \left\{ \left\{ \min_{i=1,\dots,m} p_{ik} u_{ik} \right\}_{k=1,\dots,r} : \text{ s.t. (1)–(3), (6) and } u_{ik} = 1 \right\}. \tag{9}$$

Observe that these two multiple objective optimization models treat all properties (founders) in the same way, giving each a separate objective function ($k = 1, 2, \dots, r$). It may happen that there is additional information available concerning the properties (like the genetic composition of certain founders). Then, such information may be introduced to make the treatment of individual properties (e.g., "rare" founder genes) less symmetrical. In the absence of additional information, it is suggested to retain all the symmetry present in the models.

We remark that the selection of models introduced here is not exhaustive, but rather motivated by interest in some high priority goals of managers of wildlife programs and also motivated by having capabilities to solve the models. At this point, it may not be clear that the above models are solvable for real and interesting data. That is why the contributions of the next section are important to the serious study of relocation. It will be shown, that the above primary formulations may be simplified into some nonlinear programs with a single unit knapsack constraint (all coefficients equal to one) and binary variables.

Relating back to the relocation of rare species of animals, the carriers are the animals to be relocated, and the properties are the genetic materials of the animals relative to the founders. It is not possible to differentiate these properties according to the destinations in the forest (in the wild), but it is possible to measure only those properties which do not go to destination n, which represents the zoo. This concept agrees with the idea of restoration of genetic composition of the founders, which is also independent of the destination.

For a critique of the models, notice that all are one period planning models, and all are without cost considerations relative to the actual relocation expenses. The second of these may be extremely important to overall decision making, as some destinations may be quite expensive to access. We propose a second level of optimization, which will not be discussed further in this paper. Regarding the dynamics of planning, it is an area for future research. We simply note that the collection of rare animals into zoos is an ongoing process, and so it is a feature which would necessitate a significant increase of data to build a dynamic model.

3. REFORMULATION OF MODELS

In this section, we analyze more carefully the optimization models introduced in the previous section. Let us begin with model SOSUM. Note, that due to (1)

$$\sum_{j=1}^{n-1} x_{ij} = s_i - x_{in}, \qquad \text{for } i = 1, 2 \dots, m.$$

Hence, equations (4) can be replaced by

$$y_{ik} = s_i - x_{in}, \qquad \text{for } i = 1, 2, \dots, m,\ k = 1, 2, \dots, r. \tag{10}$$

Thus, the auxiliary variables y_{ik} can be substituted into the optimization problem to yield

$$\text{SOSUM:} \qquad \max_{X} \left\{ \min_{k=1,\dots,r} \left\{ \sum_{i=1}^{m} p_{ik}(s_i - x_{in}) : \text{ s.t. (1)–(3)} \right\} \right\}. \tag{11}$$

The feasible set for problem (11) includes only standard transportation relations (1)–(3). Thus, in the case of a linear objective function it could be solved by linear programming, as integrality of decision variables would be automatically preserved (totally unimodular matrix of coefficients [2]). However, it is not true for problem (11) as the objective function is a concave nonlinear function. The objective can be easily transformed into a linear one but it requires us to introduce additional side constraints which destroy total unimodularity of the constraint matrix. Thus, problem (11) is an integer programming problem.

Note that the objective function of model SOSUM formulated with (11) depends only on decision variables x_{in} $(i = 1, 2, \ldots, m)$ and it is independent of x_{ij} for $j < n$. Relative to the conservation ecology problem, decisions related to distribution among specific destinations do not affect the "genetic" criteria. It is suggested that they should be made on the basis of another criterion on the second level after the optimization models presented here. Now, to proceed with reformulation, let us introduce a new (smaller) set of decision variables strictly related to our optimization problem. Namely,

z_i number of carriers transported from location i to one of destinations $1, 2, \ldots, n-1$, $(i = 1, 2, \ldots, m)$.

Variables z_i are related to the original decision variables x_{ij} by the following formulae:

$$z_i = s_i - x_{in}, \qquad \text{for } i = 1, 2, \ldots, m. \tag{12}$$

Hence, we can reformulate model SOSUM into an integer knapsack problem

$$\text{SOSUM: } \max_{\mathbf{z}} \left\{ \min_{k=1,\ldots,r} \left\{ \sum_{i=1}^{m} p_{ik} z_i : \sum_{i=1}^{m} z_i = d, \right.\right. \\ \left.\left. 0 \le z_i \le s_i,\ z_i \text{ integer for } i = 1, \ldots, m \right\}\right\}, \tag{13}$$

where

$$d = \sum_{j=1}^{n-1} d_j. \tag{14}$$

In order to simplify model SOBOT let us note first that problems (1)–(3) can be reformulated into one with all supplies s_i equal to 1. Such a transformation causes an increase of the number of supply points, as instead of groups of carriers at the same location, we consider each carrier independently. That means that the number of locations m is replaced with the number of all carriers $\bar{m} = \sum_{i=1}^{m} s_i$. This transformation is algebraically equivalent to replacing integer variables x_{ij} with sums of their binary components. Thus, for purpose of model SOBOT, we can replace the feasible set (1)–(3) with the following:

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad \text{for } i = 1, 2, \ldots, \bar{m}, \tag{15}$$

$$\sum_{i=1}^{\bar{m}} x_{ij} = d_j, \qquad \text{for } j = 1, 2, \ldots, n, \tag{16}$$

$$x_{ij} \in \{0, 1\}, \qquad \text{for } i = 1, 2, \ldots, \bar{m}, \quad j = 1, 2, \ldots, n. \tag{17}$$

Hence, the inequality (6) in model SOBOT can be replaced with an equation similar to (4)

$$u_{ik} = \sum_{j=1}^{n-1} x_{ij}, \qquad \text{for } i = 1, 2, \ldots, \bar{m}, \quad k = 1, 2, \ldots, r.$$

It allows us to substitute variables u_{ik}, similar to variables y_{ik} in model SOSUM, with

$$u_{ik} = 1 - x_{in}, \qquad \text{for } i = 1, 2, \ldots, \bar{m},\ k = 1, 2, \ldots, r,$$

and to form model SOBOT as follows

$$\text{SOBOT:} \qquad \max_X \left\{ \min_{k=1,\ldots,r} \left\{ \min_{i=1,\ldots,\bar{m}} (p_{ik} + p x_{in}) : \text{ s.t. } (15)\text{–}(17) \right\} \right\}, \tag{18}$$

where p is an arbitrarily large number greater than all p_{ik} (in the case of the conservation ecology problem where $0 \le p_{ik} \le 1$ one may simply put $p = 1$).

Similar to model SOSUM, the objective function in (18) depends only on decision variables x_{in} ($i = 1, 2, \ldots, \bar{m}$) and it is independent of x_{ij} for $j < n$. Thus, we can introduce a new (smaller) set of binary decision variables strictly related to our optimization problem. Namely,

z_i equal to 1 if carrier i is transported (from location i) to one of destinations $1, 2, \ldots, n-1$, and equal to 0 otherwise ($i = 1, 2, \ldots, \bar{m}$).

Binary variables z_i are related to the original decision variables x_{ij} of (15)–(17) by the formulae analogous to (12). So, the relocation problem SOBOT can be viewed as a binary knapsack problem.

$$\begin{aligned} \text{SOBOT:} \qquad \max_{\mathbf{z}} \Bigg\{ \min_{k=1,\ldots,r} \Bigg\{ & \min_{i=1,\ldots,\bar{m}} (p_{ik} - p z_i + p) : \sum_{i=1}^{\bar{m}} z_i = d, \\ & z_i \in \{0, 1\} \text{ for } i = 1, \ldots, \bar{m} \Bigg\} \Bigg\}, \end{aligned} \tag{19}$$

where d is defined with (14).

Next, we consider reformulations appropriate to the multiple objective models MOSUM and MOBOT. As above, the observations about independence relative to the destinations may be applied to achieve the simplified knapsack constraints rather than the original transportation constraints. Such simple constraints may be further exploited in various solution methods. Details about the solution techniques will be specified in the next section.

$$\begin{aligned} \text{MOSUM:} \qquad \max_{\mathbf{z}} \Bigg\{ & \left\{ \sum_{i=1}^{m} p_{ik} z_i \right\}_{k=1,\ldots,r} : \sum_{i=1}^{m} z_i = d, \\ & 0 \le z_i \le s_i,\ z_i \text{ integer for } i = 1, \ldots, m \Bigg\}, \end{aligned} \tag{20}$$

$$\begin{aligned} \text{MOBOT:} \qquad \max_{\mathbf{z}} \Bigg\{ & \left\{ \min_{i=1,\ldots,\bar{m}} (p_{ik} - p z_i + p) \right\}_{k=1,\ldots,r} : \sum_{i=1}^{\bar{m}} z_i = d, \\ & z_i \in \{0, 1\},\ \text{for } i = 1, \ldots, \bar{m} \Bigg\}. \end{aligned} \tag{21}$$

Model MOSUM is the most linear of the four and it is probably the easiest to solve, while MOBOT is in one of the toughest classes of optimization problems: nonlinear, multiple objective programs with binary variables. Very little is known about general solution algorithms for such problems. In the next section, information about solution algorithms which take advantage of special structure present in the models will be presented.

Finally, we conclude this section with Table 1 which contains the four models SOSUM, SOBOT, MOSUM and MOBOT in a compact form for easy reference and comparison. Note that both MOSUM and MOBOT models can be viewed as some kind of discrete location problems. For simplicity of the presentation, let us assume that both the models have been transformed into binary variables, i.e., to all $s_i = 1$. Then, by negating the coefficients p_{ik} or replacing them with their complements ($\bar{p}_{ik} = p - p_{ik}$), and next swapping all the max and min operators (to preserve the original sense of optimization), both the models can be expressed in the equivalent form

$$\min_{\mathbf{z}} \left\{ \{f_k(\mathbf{z})\}_{k=1,\ldots,r} : \sum_{i=1}^{m} z_i = d,\ z_i \in \{0,1\} \text{ for } i = 1,\ldots,m \right\},$$

where functions $f_k(\mathbf{z})$ are defined as

$$f_k(\mathbf{z}) = \sum_{i=1}^{m} \bar{p}_{ik} z_i \quad \text{or} \quad f_k(\mathbf{z}) = \max_{i=1,\ldots,m} \bar{p}_{ik} z_i,$$

for MOSUM and MOBOT model, respectively. So, we can view the models as discrete location problems with r clients and m potential facilities (locations), of which d facilities have to be selected for the best service of the clients. If $\bar{p}_{ik}$ depicts the distance from the facility i to the client k, the functions $f_k(\mathbf{z})$ express, respectively, the total (actually the average if divided by r) and the maximal distance of the client i to all the located facilities. It means, we consider the problem of location r different services assuming each client uses all the services and we consider two possible measures of the service quality: the average distance and the maximal distance. Each measure needs to be minimized in the corresponding problem. Further, SOSUM and SOBOT models can be viewed as center approaches to the corresponding locations problems.

Table 1. Models summary.

	Single Objective	Multiple Objective
Average	SOSUM $$\max_{\mathbf{z}} \left\{ \min_{k=1,\ldots,r} \left\{ \sum_{i=1}^{m} p_{ik} z_i : \sum_{i=1}^{m} z_i = d,\ 0 \le z_i \le s_i,\ z_i \text{ integer for } i = 1,\ldots,m \right\} \right\}$$	MOSUM $$\max_{\mathbf{z}} \left\{ \left\{ \sum_{i=1}^{m} p_{ik} z_i \right\}_{k=1,\ldots,r} : \sum_{i=1}^{m} z_i = d,\ 0 \le z_i \le s_i,\ z_i \text{ integer for } i = 1,\ldots,m \right\}$$
Worst Case	SOBOT $$\max_{\mathbf{z}} \left\{ \min_{k=1,\ldots,r} \left\{ \min_{i=1,\ldots,\bar{m}} (p_{ik} - p z_i + p) : \sum_{i=1}^{\bar{m}} z_i = d,\ z_i \in \{0,1\} \text{ for } i = 1,\ldots,\bar{m} \right\} \right\}$$	MOBOT $$\max_{\mathbf{z}} \left\{ \left\{ \min_{i=1,\ldots,\bar{m}} (p_{ik} - p z_i + p) \right\}_{k=1,\ldots,r} : \sum_{i=1}^{\bar{m}} z_i = d,\ z_i \in \{0,1\} \text{ for } i = 1,\ldots,\bar{m} \right\}$$

4. SOLUTION TECHNIQUES

All of the four models formulated in the previous section are integer knapsack problems with nonlinear objective functions. More precisely, models SOBOT and MOBOT are built with binary variables whereas models SOSUM and MOSUM with general integer variables. However, the latter can be also transformed to the form with binary variables if it would simplify the solution process.

Model SOSUM seems to be, in general, a hard integer problem. From our experience its complexity, when the branch and bound method used, strongly varied with the data. For instance, while dealing with the Przewalski's horse case study (Section 5) with $m = 191$, the complete solution with the MOMIP branch and bound code [3] the problems with d larger than r were solved in 1–40 nodes taking less than 1 CPU second (on SUN Sparc 10 workstation) whereas it took over 450000 nodes (over 15 CPU minutes) for $d = 10$.

Model SOBOT has a separable objective function. It turns out that it can be easily solved with a certain sorting procedure. Let us define for each carrier i ($i = 1, 2, \ldots, \bar{m}$) the quantities

$$p_i^* = \min_{k=1,\ldots,r} p_{ik}, \qquad \text{for } i = 1, 2, \ldots, \bar{m}. \tag{22}$$

The quantities p_i^* define optimal solution to problem (19) with the following proposition.

PROPOSITION 1. *Let quantities p_i^* be presorted in the weakly increasing order, $p_{i_1}^* \le p_{i_2}^* \le \cdots \le p_{i_{\bar{m}}}^*$, and let*

$$S_0 = \{i : p_i^* \le p_{i_d}^*\}. \tag{23}$$

Feasible vector $\bar{\mathbf{z}}$ defined by the index set S ($|S| = d$) as follows:

$$\bar{z}_i = 1 \text{ for } i \in S \quad \text{and} \quad \bar{z}_i = 0 \text{ for } i \notin S$$

is an optimal solution of model SOBOT if and only if $S \subseteq S_0$.

PROOF. Objective value for $\bar{\mathbf{z}}$ is equal to

$$\bar{p} = \min_{k=1,\ldots,r} \min_{i \in S} p_{ik} = \min_{i \in S} p_i^*.$$

Thus,

$$\bar{p} \le p_{i_d}^* \text{ for } S \subseteq S_0 \quad \text{or} \quad \bar{p} > p_{i_d}^* \text{ for } S \not\subseteq S_0.$$

Hence, $\bar{\mathbf{z}}$ is an optimal solution if and only if $S \subseteq S_0$. ∎

Note, that due to Proposition 1, in order to find an optimal solution to model SOBOT one needs only to sort the quantities p_i^* in the weakly decreasing order and next to pick up the first d carriers in the presorted sequence. Moreover, the set S_0 can be easily identified in the presorted sequence of p_i^* quantities, which allows us to find all the alternative optimal solutions (as d-element subsets of S_0).

Multiple objective model MOSUM can be easily analyzed with weighting approach, which transforms it into the following linear knapsack problem:

$$\max_{\mathbf{z}} \left\{ \sum_{i=1}^{m} \sum_{k=1}^{r} w_k p_{ik} z_i : \sum_{i=1}^{m} z_i = d,\ 0 \le z_i \le s_i \text{ for } i = 1, \ldots, m \right\}. \tag{24}$$

Due to the constraints specificity any vertex solution to this problem will satisfy the integrality requirements. It is made precise in Proposition 2.

PROPOSITION 2. *Any vertex optimal solution to problem (24) with positive weights w_k ($k = 1, 2, \ldots, r$) is an efficient solution of model MOSUM.*

PROOF. The coefficient matrix of problem (24) is totally unimodular [2]. Hence, any vertex optimal solution to this linear problem satisfies the integrality requirements and it is optimal to the corresponding integer program. Next, due to positive weights any optimal solution to the weighted integer program is an efficient solution to the corresponding multiple objective problem (compare, [4]), i.e., to MOSUM model. ∎

Note, that the linear knapsack problem (24) is, in fact, solvable with a simple sorting procedure. The algorithms in [5, Chapter 4] are of low complexity and they are recommended if finding a single nondominated solution is desired. In order to find a vertex optimal solution to the problem, one needs only to sort the quantities

$$p_i^w = \sum_{k=1}^{r} w_k p_{ik}, \qquad \text{for } i = 1, 2, \ldots, m,$$

in the weakly decreasing order and next to pick up the first d carriers in the presorted sequence. Weights inversely proportional to the totals of several properties (founder contributions in the conservation ecology problem) seem to be especially interesting [6].

Note, that model MOSUM may be reformulated into that with all $s_i = 1$ (as we did with SOBOT and MOBOT models). In this case, due to the constraints specificity, any vertex feasible solution is an integer one, and any integer solution is a vertex one. So, while dealing with a single linear objective we could simply forget about integrability requirements and solve the corresponding linear problem. Unfortunately, with multiple linear criteria it is not so easy. Although every integer efficient solution is a vertex solution and every vertex solution is an integer one, there may exist an integer efficient solution which is no longer efficient if considered in the linear problem (a vertex nondominated within the set of vertices but dominated by some nonvertex solution). Thus, even in the case of all $s_i = 1$, the weighting approach does not provide us with a complete parametrization of the entire efficient set. As an example one may consider a problem with two properties P1, P2 ($r = 2$), and three carriers C1, C2, C3 ($m = 3$) of which only need to be selected to move ($d = 1$). Due to d equal to 1, one may certainly assume that $s_i = 1$ for $i = 1, 2, 3$. Let the coefficients p_{ik} ($i = 1, 2, 3$, $k = 1, 2$) be given with the following table:

	P1	P2
C1	0	9
C2	9	0
C3	4	4

Note, that all three feasible solutions, i.e., selection of any carrier, are efficient solutions. One can easily verify that while dealing with weighting approach to the problem, carrier C3 (despite being a very attractive compromise solution) cannot be selected for any set of positive weights assigned to clients. If P1 has been assigned the higher weight than P2 ($w_1 > w_2$), then carrier C2 is a unique optimal solution to the weighted problem. If P1 has been assigned the lower weight than P2 ($w_1 < w_2$), then carrier C1 is a unique optimal solution to the weighted problem. Finally, if both properties have been assigned equal weights ($w_1 = w_2$), then both carriers C1 and C2 are optimal.

In our specific relocation problem related to the conservation ecology it was not a difficulty, as the coefficients p_{ik} satisfied relations

$$\sum_{k=1}^{r} p_{ik} = 1, \qquad \text{for } i = 1, 2, \ldots, m.$$

In such a case, every efficient solution of MOSUM model is an optimal solution to the linear problem (24) with equal weights ($w_1 = w_2 = \cdots = w_r$). In a general case, another multiple objective approach may be used to allow us to find any efficient solution. For instance, while using generating techniques based on the weighted Chebyshev distance [4], we get the parametrized SOSUM model to solve. For finding all nondominated solutions, it is possible to use multiple objective dynamic programming.

Multiple objective model MOBOT may be easily analyzed with approaches based on generating techniques using the weighted Chebyshev distance, like the reference point method [4,7]. When, we specify for each property an aspiration level a_k and weight w_k; then in order to generate an efficient solution, we need to solve the following single objective problem:

$$\max_{\mathbf{z}} \left\{ \min_{k=1,\dots,r} \left\{ \min_{i=1,\dots,\bar{m}} w_k(p_{ik} - a_k - pz_i + p) : \sum_{i=1}^{\bar{m}} z_i = d, \right.\right.$$
$$\left.\left. z_i \in \{0,1\} \text{ for } i = 1,\dots,\bar{m} \right\}\right\}. \tag{25}$$

Problem (25) differs from model SOBOT only due to weighting and shifting of coefficients p_{ik}. It can be considered as a parametrization of model SOBOT. Thus, similarly as the latter, it can be easily solved by sorting.

PROPOSITION 3. *For any $\bar{\mathbf{z}}$ efficient solution of MOBOT there exist aspiration levels a_k and positive weights w_k such that $\bar{\mathbf{z}}$ is an optimal solution to problem (25). For any aspiration levels a_k and positive weights w_k, a unique optimal solution to problem (25) is an efficient solution of model MOBOT.*

PROOF. In order to prove the first statement, let us define aspiration levels

$$a_k = \min_{i \in S} p_{ik},$$

where S is the index set defining the efficient solution $\bar{\mathbf{z}}$. With such aspiration levels, for any positive weights w_k, $\bar{\mathbf{z}}$ is an optimal solution to problem (25).

Now, let us consider $\bar{\mathbf{z}}$ as a unique optimal solution of the problem (25) with some aspiration levels a_k and positive weights w_k. Suppose it is not an efficient solution of model MOBOT. It means, there exists a feasible solution $\mathbf{z}$ such that

$$\min_{i=1,\dots,\bar{m}} (p_{ik} - pz_i + p) \geq \min_{i=1,\dots,\bar{m}} (p_{ik} - p\bar{z}_i + p), \qquad \text{for } k = 1,2,\dots,r.$$

Hence,

$$\min_{i=1,\dots,\bar{m}} w_k(p_{ik} - a_k - pz_i + p) \geq \min_{i=1,\dots,\bar{m}} w_k(p_{ik} - a_k - p\bar{z}_i + p), \qquad \text{for } k = 1,2,\dots,r,$$

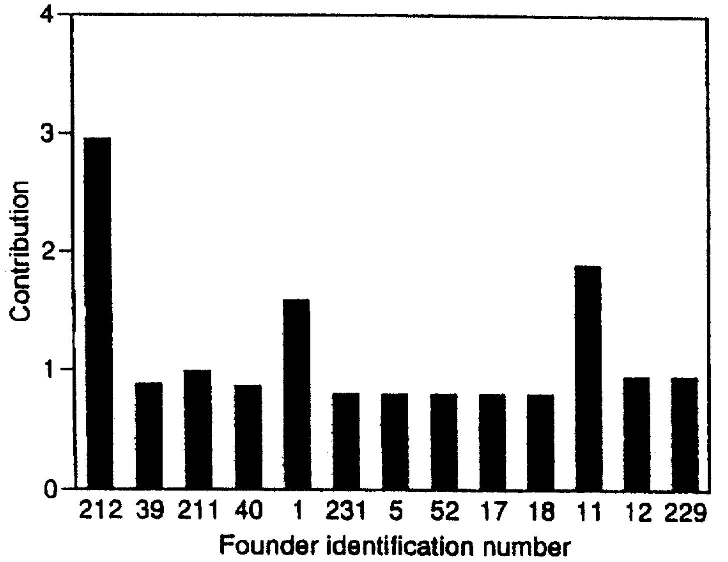
which contradicts our assumption that vector $\bar{\mathbf{z}}$ is a unique optimal solution of problem (25). ∎

In the case of nonunique optimal solution to problem (25), some regularization techniques are necessary to guarantee that the efficient solution will be selected [8,9]. Typically some weighted terms are used for the regularization. Such a regularization would destroy the maximin structure of the problem (25). Therefore, we are rather interested in a lexicographic (or nucleolar) regularization [10,11]. The lexicographic form of problem (25) can be solved by sequential optimization of problems (25) with especially modified objective functions [12]. The same approach may be used to the standard SOBOT model to refine the selection among the alternative optimal solutions.

5. CASE STUDY

The example we present here is useful to illustrate the capabilities of the models developed as applied to real (or realistic) data. The data we used was that of the Przewalski's horse herd. There were considered 191 living animals (North American Population, as of December 31, 1992), which could be traced to 13 founder horses. The totals of founders genetic contributions are plotted in Figure 1.

In studying the Przewalski's horse data, we solved several relocation problems. Both MOSUM and MOBOT models were handled whereas SOSUM and SOBOT models were considered as



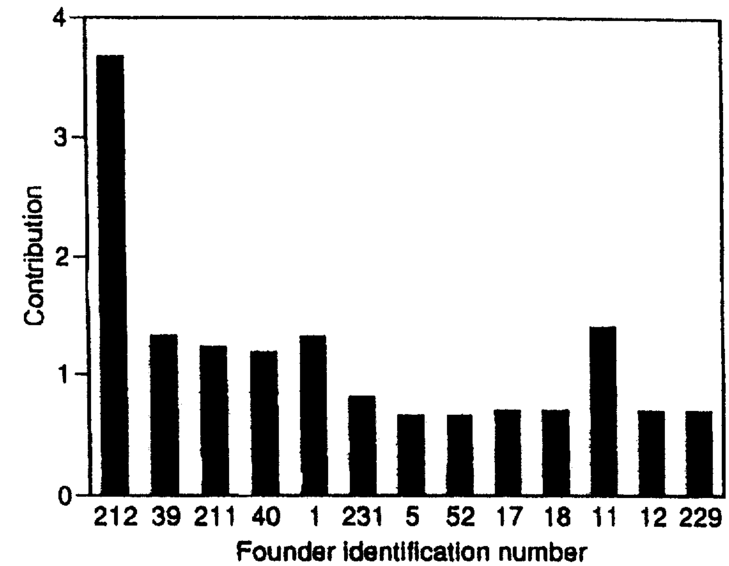
	Founder Identificaiton Number												
	212	39	211	40	1	231	5	52	17	18	11	12	229
Descendant ID#													
535	0.375	0.031	0.125	0.031	0.188	0	0.094	0.094	0.031	0.031	0	0	0
615	0	0.094	0	0.094	0	0	0	0	0.094	0.094	0.313	0.156	0.156
667	0.117	0.078	0.039	0.078	0.063	0.063	0.031	0.031	0.078	0.078	0.172	0.086	0.086
694	0.188	0.047	0.063	0.047	0.125	0	0.063	0.063	0.047	0.047	0.156	0.078	0.078
700	0.305	0.024	0.102	0.024	0.188	0.063	0.094	0.094	0.024	0.024	0.031	0.016	0.016
718	0.234	0.047	0.078	0.047	0.125	0.125	0.063	0.063	0.047	0.047	0.063	0.031	0.031
886	0.059	0.078	0.02	0.078	0.031	0.031	0.016	0.016	0.078	0.078	0.258	0.129	0.129
1144	0.188	0.055	0.063	0.055	0.125	0	0.063	0.063	0.055	0.055	0.141	0.07	0.07
1166	0.234	0.047	0.078	0.047	0.125	0.125	0.063	0.063	0.047	0.047	0.063	0.031	0.031
1167	0.176	0.055	0.059	0.055	0.094	0.094	0.047	0.047	0.055	0.055	0.133	0.067	0.067
1174	0.117	0.063	0.039	0.063	0.063	0.063	0.031	0.031	0.063	0.063	0.203	0.102	0.102
1185	0.176	0.063	0.059	0.063	0.094	0.094	0.047	0.047	0.063	0.063	0.117	0.059	0.059
1408	0.375	0.043	0.125	0.035	0.203	0	0.102	0.102	0.008	0.008	0	0	0
5019	0.138	0.079	0.046	0.076	0.059	0.047	0.029	0.029	0.066	0.066	0.184	0.092	0.092
5050	0.27	0.074	0.09	0.067	0.109	0.094	0.055	0.055	0.039	0.039	0.055	0.027	0.027
Sums:	2.951	0.876	0.984	0.858	1.59	0.797	0.795	0.795	0.793	0.793	1.887	0.944	0.944

Figure 2. Sosum: Single Objective-Sum. Selection of 15 P-horses that maximizes the ninimum (unweighted) summed founder contribution.

special cases of the multiple criteria analysis. In both multiple criteria models, we decided to use certain weights. Weights which seem to be very interesting are the inverse total founder contributions. Such a set of weights will put more emphasis on the rare genes of the given founders. We have also tried the inverse maximal founder contribution weights and unit weights (all weights equal to one).

While analyzing the MOBOT model, we used the reference point approach [7], thus getting the weighted SOBOT problems (25) to be solved with the sorting procedure (Proposition 1).

While analyzing MOSUM model, we used two approaches: the reference point method and the linear weighting approach. When using the reference point method, we got the weighted SOSUM problems to be solved. As these were general MIP problems, we solved them with a branch and bound code. Using the MOMIP code [3], the problems which interested us the most (i.e., with $d = 15 - 20$) were solved very quickly in less than 1 CPU second (on SUN Sparc 10 workstation).



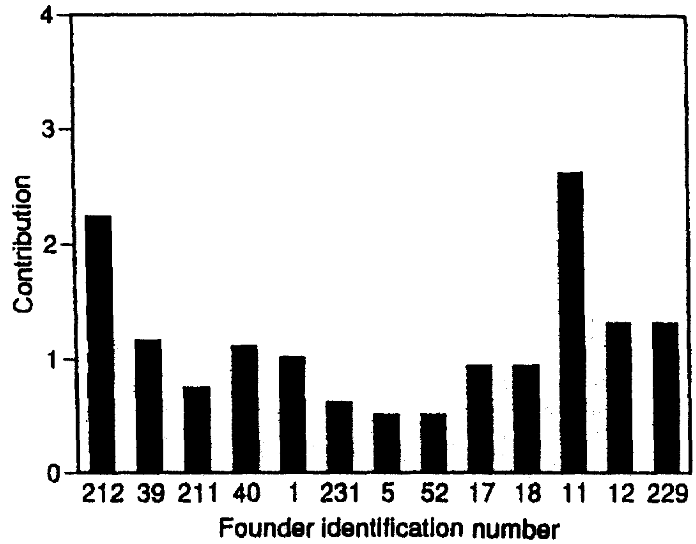
	Founder Identification Number												
	212	39	211	40	1	231	5	52	17	18	11	12	229
Descendant ID#													
1167	0.176	0.055	0.059	0.055	0.094	0.094	0.047	0.047	0.055	0.055	0.133	0.067	0.067
1185	0.176	0.063	0.059	0.063	0.094	0.094	0.047	0.047	0.063	0.063	0.117	0.059	0.059
5052	0.211	0.082	0.07	0.074	0.078	0.063	0.039	0.039	0.047	0.047	0.125	0.063	0.063
5051	0.281	0.099	0.094	0.085	0.086	0.047	0.043	0.043	0.037	0.037	0.074	0.037	0.037
5049	0.281	0.099	0.094	0.085	0.086	0.047	0.043	0.043	0.037	0.037	0.074	0.037	0.037
5022	0.264	0.068	0.088	0.062	0.117	0.063	0.059	0.059	0.041	0.041	0.07	0.035	0.035
1806	0.264	0.068	0.088	0.062	0.117	0.063	0.059	0.059	0.041	0.041	0.07	0.035	0.035
1287	0.258	0.096	0.086	0.084	0.078	0.031	0.039	0.039	0.043	0.043	0.102	0.051	0.051
1400	0.258	0.104	0.086	0.092	0.078	0.031	0.039	0.039	0.051	0.051	0.086	0.043	0.043
1514	0.258	0.096	0.086	0.084	0.078	0.031	0.039	0.039	0.043	0.043	0.102	0.051	0.051
1515	0.258	0.104	0.086	0.092	0.078	0.031	0.039	0.039	0.051	0.051	0.086	0.043	0.043
1587	0.258	0.096	0.086	0.084	0.078	0.031	0.039	0.039	0.043	0.043	0.102	0.051	0.051
1673	0.246	0.125	0.082	0.11	0.063	0.031	0.031	0.031	0.055	0.055	0.086	0.043	0.043
1713	0.246	0.117	0.082	0.102	0.063	0.031	0.031	0.031	0.047	0.047	0.102	0.051	0.051
718	0.234	0.047	0.078	0.047	0.125	0.125	0.063	0.063	0.047	0.047	0.063	0.031	0.031
Sums:	3.669	1.316	1.224	1.179	1.313	0.813	0.657	0.657	0.701	0.701	1.392	0.696	0.696
Available	63.61	22.11	21.21	18.7	15.72	10.67	7.868	7.868	6.763	6.763	4.93	2.468	2.468

Figure 3. Sobot: Single Objective-Bottleneck. Selection fo 15 P-horses that maximizes the minimum founder contribution to any single animal.

However, we have found out that for some values of d (smaller than r), the branch and bound process may take as long as 15 CPU minutes.

To illustrate the results of our computations, we present bar graphs of founder values for the particular models, all for $d = 15$. Figure 2 contains the results of MOSUM/SOSUM (unit weights), while Figure 3 contains MOBOT/SOBOT (unit weights) and Figure 4 represents the MOSUM/SOSUM information (inverse total founder contribution weights). For an appreciation of the leveling effect of the models's optimizations, the reader should compare these graphs with Figure 1.

When using the linear weighting approach, we got linear knapsack problems (24) solved with the simple sorting procedure. In this approach, the case of unit weights was not considered as in that case every feasible solution is optimal. Thus, we have limited this analysis to weights



	Founder Identification Number												
	212	39	211	40	1	231	5	52	17	18	11	12	229
Descendant ID#													
615	0	0.094	0	0.094	0	0	0	0	0.094	0.094	0.313	0.156	0.0156
667	0.117	0.078	0.039	0.078	0.063	0.063	0.031	0.031	0.078	0.078	0.172	0.086	0.086
668	0.117	0.063	0.039	0.063	0.063	0.063	0.031	0.031	0.063	0.063	0.203	0.102	0.102
694	0.188	0.047	0.063	0.047	0.125	0	0.063	0.063	0.047	0.047	0.156	0.078	0.078
886	0.059	0.078	0.02	0.078	0.031	0.031	0.016	0.016	0.078	0.078	0.258	0.129	0.129
1144	0.188	0.055	0.063	0.055	0.125	0	0.063	0.063	0.055	0.055	0.141	0.07	0.07
1167	0.176	0.055	0.059	0.055	0.094	0.094	0.047	0.047	0.055	0.055	0.133	0.067	0.067
1174	0.117	0.063	0.039	0.063	0.063	0.063	0.031	0.031	0.063	0.063	0.203	0.102	0.102
1185	0.176	0.063	0.059	0.063	0.094	0.094	0.047	0.047	0.063	0.063	0.117	0.059	0.059
1587	0.258	0.096	0.086	0.084	0.078	0.031	0.039	0.039	0.043	0.043	0.102	0.051	0.051
5019	0.138	0.079	0.046	0.076	0.059	0.047	0.029	0.029	0.066	0.066	0.184	0.092	0.092
5020	0.15	0.09	0.05	0.085	0.054	0.035	0.027	0.027	0.067	0.067	0.175	0.088	0.088
5024	0.173	0.11	0.058	0.099	0.043	0.016	0.022	0.022	0.062	0.062	0.168	0.084	0.084
5025	0.173	0.11	0.058	0.099	0.043	0.016	0.022	0.022	0.062	0.062	0.168	0.084	0.084
5052	0.211	0.082	0.07	0.074	0.078	0.063	0.039	0.039	0.047	0.047	0.125	0.063	0.063
Sums:	2.24	1.161	0.747	1.112	1.011	0.614	0.506	0.506	0.94	0.94	2.617	1.309	1.309
Weighted:	0.035	0.053	0.035	0.059	0.064	0.058	0.064	0.064	0.139	0.139	0.531	0.531	0.531

Figure 4. Mosum: Multiple Objective-Sum. Selection of 15 P-Horses that simultaneously maximizes all (weighted) summed founder contributions. Founders weighted inversely by their total contribution.

defined as the inverse total or maximal founder contribution. Due to the solution procedure specificity, with this approach it was possible to make a study of the percentage change in the objective function with respect to changing the right-hand side. This is of interest because the manager never knows exactly how many animals to relocate. So, it might be desirable to continue relocating animals until the marginal gain is zero or negligible. For the data represented by the Przewalski's horse population and the inverse total founders contribution as the weights, the percent marginal gain in objective values behaved as in Table 2.

For our modeling and analysis of Przewalski's horse North American Population, we used all animals available. It would be also possible to model with subpopulations based on age ranges (say three to four year old animals) or other characteristics. For larger applications this may, in fact, be a requirement for some of the models. It is demonstrated by the application at hand that

Table 2. Percent marginal gain in objective values for MOSUM weighted with the inverse total founders contributions.

Number Relocated	Marginal Gain(%)	Number Relocated	Marginal Gain(%)
10	8.21	100	0.64
15	4.75	105	0.62
20	3.62	110	0.59
25	2.92	115	0.55
30	2.48	120	0.54
35	2.00	125	0.52
40	1.71	130	0.51
45	1.55	135	0.50
50	1.25	140	0.47
55	1.18	145	0.46
60	1.05	150	0.44
65	0.97	155	0.43
70	0.91	160	0.42
75	0.85	165	0.41
80	0.80	170	0.40
85	0.76	175	0.37
90	0.71	180	0.36
95	0.67	185	0.34
		190	0.29

reasonably sized models can be solved by the approaches described in this paper, with readily available computer hardware and software.

6. CONCLUSIONS

This paper introduces and solves several mathematical optimization models connected with the concept of relocation. For these models, it is demonstrated that the data needed and the corresponding solution methods are at hand, ready for large-scale realistic applications. Motivation for the study comes from the biological problem of the management of rare or endangered animal species with the goal of restoration of the genetic composition of founder populations. It is felt that an interdisciplinary effort such as that employed here is likely to produce significant applications of these ideas in related but disconnected areas of study.

Contained here is a progression of models, all equivalent in terms of optimal solutions, but very different in structure and in their solution techniques. Models which are finally solved are not natural for direct formulation, while those which come to mind in the primary formulation appear to be too difficult to solve. Thus, the paper serves as a strong illustration of the need for human mathematical reasoning skills together with the utilization of modern computational resources all applied to solving a well-motivated problem outside of what is considered the normal realm of operations research or applied mathematics. Biologists appreciate the way in which their wildlife management goals may be quantified and, in fact, achieved in an optimal solution. For operations research specialists, what may be most interesting is the simplicity with which some seemingly difficult integer and multiple objective programming problems may be solved. From the point of view of those interested specifically in location theory, it is a contribution which opens a new problem area, while pointing the way to great potential applications in more general planning environments.

REFERENCES

1. O.A. Ryder, Przewalski's horse: Prospects for reintroduction into the wild, *Conservation Biology* **7**, 13–15 (1993).
2. G.L. Nemhauser and L.A. Wolsey, *Integer and Combinatorial Optimization*, John Wiley, New York, (1988).

3. W. Ogryczak and K. Zorychta, Modular optimizer for mixed integer programming, MOMIP Version 2.1. WP-94-035, IIASA, Laxenburg, (1994).
4. R.E. Steuer, *Multiple Criteria Optimization—Theory, Computation and Applications*, John Wiley, New York, (1986).
5. T. Ibaraki and N. Katoh, *Resource Allocation Problems, Algorithmic Approaches*, MIT Press, Cambridge, MA, (1988).
6. D.L. Bannister, An operations research approach to endangered species relocation, MS Project, Department of Mathematical Sciences, Clemson University, Clemson, SC, (1992).
7. A.P. Wierzbicki, A mathematical basis for satisfying decision making, *Mathematical Modelling* **3**, 391–405 (1982).
8. A.P. Wierzbicki, On the completeness and constructiveness of parametric characterization to vector optimization problems, *OR-Spectrum* **8**, 73–87 (1986).
9. W. Ogryczak and S. Lahoda, Aspiration/reservation based decision support—A step beyond goal programming, *Journal of Multi-Criteria Decision Analysis* **1**, 101–117 (1992).
10. E. Marchi and J.A. Oviedo, Lexicographic optimality in the multiple objective linear programming: The nucleolar solution, *European Journal of Operational Research* **57**, 355–359 (1992).
11. J.A.M. Potters and S.H. Tijs, The nucleolus of a matrix game and other nucleoli, *Mathematics of Operations Research* **17**, 164–174 (1992).
12. M. Maschler, J.A.M. Potters and S.H. Tijs, The general nucleolus and the reduced game property, *International Journal of Game Theory* **21**, 85–106 (1992).
13. G. Yu and P. Kouvelis, Complexity results for a class of min-max problems with robust optimization applications, In *Complexity in Numerical Optimization*, (Edited by P. Pardalos), pp. 501–511, World Scientific, Singapore, (1993).