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Equitable goal programming model for periodic vehicle routing and scheduling problems

Włodzimierz Ogryczak, Tomasz Śliwiński, Bartosz Kozłowski
Institute of Control and Computation Engineering, Warsaw University of Technology
00-665 Warsaw, Poland
{W.Ogryczak,T.Sliwinski,B.Kozlowski}@elka.pw.edu.pl

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1 Introduction

Decision support system for planning and management of mobile personnel tasks has to deal with complex optimization problems. The SATIS software [10] supports various fleet management tasks as well as management of work of a sales team and allows to stimulate sales activity. The latter functions include planning of visits and meetings of field employees, assignment of tasks to be performed and preparation and settlement of sales plans. Daily access to data is ensured by embedded indicators and reports such as, for example, planning of work of commercial representatives, monitoring of execution of a plan or report on visits. Gathered data is used to measure effectiveness of employees, allows to define objectives and motivates to improve sales results. SATIS Optimisation module gives a possibility to set the most advantageous route so that a driver may cover a distance in the appropriate time. Optimization of processes reduces fuel consumption and travel time, due to which mobile employees may work more effectively, maintaining or reducing costs. For efficient support of planning and management of mobile personnel tasks there have been developed procedures [8] to deal with large-scale periodic time-dependent vehicle routing and scheduling problems with complex nonuniform constraints with respect to frequency, time windows, working time, etc. Such a problem might be considered as a generalization of the Time-Dependent Traveling Salesman Problem (TDTSP) [1, 2, 3, 13] with several real features, specifically the following: each point (customer) can define multiple time windows during which it is available and can be serviced; the travel time between the points varies, due to the traffic, and actually it depends on the traffic time zone, in which the transit actually occurs; starting and ending depots are treated as points so that they also have time windows. Moreover, additional fast adaptive procedures for operational rescheduling of plans in presence of various disturbances are needed. Several solution quality indicators with respect to a single personnel person must be considered.

2 Equitable goal programming model

The objectives are structured in a two-level lexicographic optimization where on the top level is maximized the number of visits completed according to all the restrictions. At the bottom level four quantified schedule quality criteria are aggregated: minimized travel cost, minimized labor cost (including overtime costs), minimized excess deviation from the weekly working time norm, minimized lower and upper deviations from the reference visit frequency (gap between subsequent visits). All the second level objectives are defined as the goal programming deviational criteria with penalty functions [9] and unattainable targets. Thus they built a GP model though equivalent to the Reference Point Method [4].

Since, even single-objective problem is extremely difficult to solve, due to the number of binary variables, for medium and larger number of customers it is more efficient to tackle it with heuristic approaches. Business requirements enforce that in short time (operational) perspective every point is visited by the same personnel person. In longer time horizon, due to optimization needs or personnel fluctuation, there is possible a change of the personnel allocation. Therefore, while solving the problem one can separate the phase of personnel allocation to points and simultaneous allocation of visits to specific days with respect to required frequency and personnel limitations. The multicriteria preference model is adjusted already during the first stage. Approximate solution depends on the use some metaheuristic to examine various sequences of points while comparing the objective function values. Particularly, we have used the list based threshold accepting heuristic [11]. For comparison the objective values, there is no problem with handling two-level optimization. Although, the second level objectives must be scalarized into one achievement function.

An equitable aggregation of the individual achievement functions is needed. Max-min aggregation is typically used in GP for this purpose. It requires a regularization to guarantee efficiency with respect to all individual achievements optimization. The regularization by the average achievement is easily implementable but it may disturb the basic max-min model. Actually, the only consequent regularization of the max-min aggregation is the lex-min order or more practical the OWA aggregation with monotonic weights [5]. The latter combines all the partial achievements allocating the largest weight to the worst achievement, the second largest weight to the second worst achievement, the third largest weight to the third worst achievement, and so on. The OWA aggregation [14] is mathematically formalized as follows. Within the space of achievement vectors we introduce map $\Theta = (\theta_1, \theta_2, \ldots, \theta_m)$ which orders the coordinates of achievements vectors in a nondecreasing order, i.e., $\Theta(a_1, a_2, \ldots, a_m) = (\theta_1(\mathbf{a}), \theta_2(\mathbf{a}), \ldots, \theta_m(\mathbf{a}))$ iff there exists a permutation τ such that $\theta_i(\mathbf{a}) = a_{\tau(i)}$ for all i and $\theta_1(\mathbf{a}) \leq \theta_2(\mathbf{a}) \leq \ldots \leq \theta_m(\mathbf{a})$. The standard max-min aggregation depends on maximization of $\theta_1(\mathbf{a})$ and it ignores values of $\theta_i(\mathbf{a})$ for $i \geq 2$. OWA represents the weighted combination of of the ordered achievements. The weights are then assigned to the specific positions within the ordered achievements rather than to the partial achievements themselves:

$$\max \sum_{i=1}^{m} w_i \theta_i(\mathbf{a}) \tag{1}$$

where $w_1 > w_2 > \ldots > w_m > 0$ are positive and strictly decreasing weights. Actually, they should be significantly decreasing to represent regularization of the max-min order. Aggregating only four achievement functions ($m = 4$), there could be easily defined strictly decreasing weights $w_i = 10^{-2(i-1)}$. (They need not to be normalized to sum up to 1).

Due to different importance of several achievements related to various preferences, there is a need for some importance weights control. Typical Min-Max aggregations allow weighting of several achievements only by straightforward rescaling of the achievement values. The OWA model enables one to introduce importance weights $\mathbf{v} = (v_1, \ldots, v_m)$ such that $v_i \geq 0$ for $i = 1, \ldots, m$ as well as $\sum_{i=1}^{m} v_i = 1$ to affect achievement importance by rescaling accordingly its measure within the distribution of achievements as defined in [7, 6]. Such a scalarization is defined by modification of formula (1) where the OWA weights w_i are applied to averages of the corresponding uniform portions of ordered achievements (quantile intervals) according to the distribution defined by importance weights v_i. That is [7]:

$$A_{\mathbf{w},\mathbf{v}}(\mathbf{a}) = \sum_{i=1}^{m} w_i m \int_{\frac{i-1}{m}}^{\frac{i}{m}} F_{\mathbf{a}}^{(-1)}(\xi)\; d\xi \tag{2}$$

where $F_{\mathbf{a}}^{(-1)}$ is the stepwise function $F_{\mathbf{a}}^{(-1)}(\xi) = \theta_i(\mathbf{a})$ for $i - 1/m < \xi \leq i/m$. It can also be mathematically formalized as follows. First, we introduce the right-continuous cumulative distribution function (cdf):

$$F_{\mathbf{a}}(d) = \sum_{i=1}^{m} v_i \delta_i(d) \tag{3}$$

where $\delta_i(d) = 1$ if $a_i \leq d$ and 0 otherwise. Next, we introduce the quantile function $F_{\mathbf{a}}^{(-1)}$ as the left-continuous inverse of the cumulative distribution function $F_{\mathbf{a}}$, ie., $F_{\mathbf{a}}^{(-1)}(\xi) = \inf\ \{\eta : F_{\mathbf{a}}(\eta) \geq \xi\}$ for $0 < \xi \leq 1$. Note that formula (2) is equivalent to the standard OWA (1) in the case of equal importance weights ($v_i = 1/m$ for $i = 1, \ldots, m$) as well as it covers the standard weighted mean with importance weights v_i as a special case of equal OWA weights ($w_i = 1/m$ for $i = 1, \ldots, m$).

Formula (2) may be reformulated with the tail averages thus leading to an LP implementable form of the scalarization [6]. Although, due to the use of the heuristic algorithm requiring only calculations and comparisons of the scalarizing function values, LP implementability of the formula is not important. More effective computational formula can be then applied, instead. Taking advantages of the finite number of steps in function $F_{\mathbf{a}}^{(-1)}$ and their correspondence to values $\theta_i(\mathbf{a})$, the scalarization (2) may be expressed, similar to the original OWA (1), as the weighted combination the ordered values although with apropriately recalculated weights:

$$A_{\mathbf{w},\mathbf{v}}(\mathbf{a}) = \sum_{i=1}^{m} \omega_i \theta_i(\mathbf{a}) \tag{4}$$

where the weights ω_i are defined as

$$\omega_i = w^*(\sum_{k \leq i} v_{\tau(k)}) - w^*(\sum_{k<i} v_{\tau(k)}) \tag{5}$$

with τ representing the ordering permutation for $\mathbf{a}$ (i.e. $a_{\tau(i)} = \theta_i(\mathbf{a})$) and the piece-wise linear increasing function w^*:

$$w^*(\xi) = \begin{cases} m w_1 \xi & \text{for } 0 \leq \xi \leq \dfrac{1}{m} \\ \displaystyle\sum_{j=1}^{k} w_j + m w_k (\xi - \frac{k}{m}) & \text{for } \dfrac{k}{m} < \xi \leq \dfrac{k+1}{m},\ k = 1, \ldots, m-1 \end{cases} \tag{6}$$

As function w^* defined by (6) interpolates points $(\frac{i}{m}, \sum_{k \leq i} w_k)$ together with the point (0.0), the scalarization formula (4) fits the general formula for the so-called Weighted OWA (WOWA) aggregation [12]. The formula (6) depends only on the the OWA weights w_i. Hence, it is uniquely defined for the entire process while only formulae (4) and (5) have to be recalculated for various solutions. Moreover, the control parameters represented by importance weights v_i affect only the formula (5).

In summary, the multiple objectives have been structured in the lexicographic optimization with the function f_0 at the top level and the ordered weighted average of the remaining four criteria with importance weights (5). The latter defines the control parameters for the preference model. Since do to business requirements while solving the problem it has been separated the initial phase of personnel allocation to points and simultaneous allocation of visits to specific days with respect to required frequency and personnel limitations. The importance weights (preference model) have been adjusted already during this initial stage of the analysis.

3 Concluding remarks

Supporting for planning and management of mobile personnel tasks requires solving large-scale periodic time-dependent vehicle routing and scheduling problems with complex constraints and goals. Several solution quality indicators with respect to a single personnel person have to be considered. In the developed system the objectives are structured in a two-level lexicographic optimization where on the top level there is maximized the number of visits completed according to all the restrictions. At the bottom level four quantified schedule quality criteria are defined as the goal programming deviational achievement functions. They are aggregated into ordered weighted average with importance weights. The latter represent control parameters while adjusting the aggregation to the decision maker preferences. The approach is applicable for hard discrete optimization problem solved with threshold accepting metaheuristics.

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