Predictive controllers integrated with economic optimization in constrained control systems tolerating sensor faults

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Introduction – *predictive algorithms integrated with economic optimization*

- One of the methods to cope with disturbances changing quickly comparing to the dynamics of the control plant
- The steady–state control plant model is linearized
- The control system structure is simplified
- Only one quadratic optimization problem must be solved at each iteration
- The economic optimization is performed more often than in the classic hierarchical approach
Introduction – fault tolerance

• Continuation of the control system operation till the failure is fixed
• The loss of measurement means the interruption of the feedback loop
  — unstable operating point: guide the process to the region of safe operation
  — stable operating point: continue operation in the acceptable way
• Output constraints are often important for process safety and its economic effectiveness
The idea of the predictive control

Fig. 1. Idea of predictive control; \( p \) – prediction horizon, \( s \) – control horizon, \( \Delta u_k \) – control signal change at current iteration
Numerical predictive control algorithms

Following problem is solved at each iteration:

$$\min_{\Delta u} \left\{ J_{MPC} = \sum_{j=1}^{n_y} \sum_{i=1}^{p} \kappa_j \cdot (\bar{y}_k^j - y_{k+il}^j)^2 + \sum_{j=1}^{n_u} \sum_{i=0}^{s-1} \lambda_j \cdot (\Delta u_{k+il}^j)^2 \right\}$$

subject to the constraints:

$$\Delta u_{\min} \leq \Delta u \leq \Delta u_{\max},$$

$$u_{\min} \leq u \leq u_{\max},$$

$$y_{\min} \leq y \leq y_{\max},$$
**Numerical predictive control algorithms**

Following problem is solved at each iteration:

\[
\min_{\Delta u} \left\{ J_{MPC} = (\bar{y} - y)^T \cdot \kappa \cdot (\bar{y} - y) + \Delta u^T \cdot \lambda \cdot \Delta u \right\}
\]

subject to the constraints:

\[
\Delta u_{\text{min}} \leq \Delta u \leq \Delta u_{\text{max}}, \quad u_{\text{min}} \leq u \leq u_{\text{max}}, \quad y_{\text{min}} \leq y \leq y_{\text{max}},
\]

- In a nonlinear case, in order to avoid problems connected with general nonlinear optimization, effective algorithms with model linearization and quadratic optimization are used.
- A few such algorithms are available, so the algorithm most suitable for a given nonlinear plant can be selected and a compromise between control performance and computation demand can be achieved.
Economic optimization problem

\[
\min_{\bar{y}} J_E(\bar{y}, \bar{u})
\]

subject to

\[
\bar{u}_{\min} \leq \bar{u} \leq \bar{u}_{\max}
\]

\[
\bar{y}_{\min} + \bar{r}_{\min} \leq \bar{y} \leq \bar{y}_{\max} - \bar{r}_{\max}
\]

\[
\bar{y} = F(\bar{u}, \tilde{w})
\]

\(F : \mathbb{R}^{n_u} \times \mathbb{R}^{n_w} \rightarrow \mathbb{R}^{n_y}\) is a steady–state plant model (\(u\) – inputs, \(y\) – outputs, \(w\) – disturbances)

- Precise nonlinear steady–state plant model
Predictive algorithms integrated with economic optimization

\[
\begin{align*}
\min_{\Delta u, \bar{y}} & \quad J_{\text{MPC}}(\bar{y}, \Delta u) + \gamma \cdot J_E(\bar{y}, \bar{u}) \\
\text{subject to} & \\
\Delta u_{\text{min}} & \leq \Delta u \leq \Delta u_{\text{max}}, \\
u_{\text{min}} & \leq u \leq u_{\text{max}}, \\
y_{\text{min}} & \leq y \leq y_{\text{max}}, \\
\bar{u}_{\text{min}} & \leq \bar{u} \leq \bar{u}_{\text{max}} \\
\bar{y}_{\text{min}} + \bar{r}_{\text{min}} & \leq \bar{y} \leq \bar{y}_{\text{max}} - \bar{r}_{\text{max}} \\
\bar{y} & = F(u(k-1), \tilde{w}) + H(k)(\bar{u} - u(k-1))
\end{align*}
\]

- Constraints from economic optimization problem
- Economic optimization performance function
- Linearization of the steady-state nonlinear model \(F\)
Basic approach to sensor fault accommodation

- Control the loop in which the fault occurred in the open-loop configuration (feedforward control)
  - In practice: calculation of the free response using predicted instead of measured value of the output with damaged measurement
  - The problem: the disturbances acting on the control plant will not be compensated on the output with broken measurement

- Use of the disturbance measurements is crucial
The case of constraints put on output variable values

- Change (shift) of constraints in the predictive controller

\[ y_{\text{min}} + r_{\text{min}} \leq y \leq y_{\text{max}} - r_{\text{max}} \]
The case of constraints put on output variable values

- Change (shift) of constraints in the predictive controller

\[ y_{\text{min}} + r_{\text{min}} - \tilde{y} \leq A \cdot \Delta u \leq y_{\text{max}} - r_{\text{max}} - \tilde{y} \]

\( \tilde{y} \) is a free response

\( A \) is a dynamic matrix

- Problem of an empty set of admissible solutions may occur

- Mechanism of soft constraints can cause violation of the constraints
The case of constraints put on output variables

- Modification of the constraints influencing the set–point values:

\[ \bar{y}_{\min} + \bar{r}_{\min} \leq \bar{y} \leq \bar{y}_{\max} - \bar{r}_{\max} \]

for the lower bound:

\[ \bar{r}_{\min}^j = \bar{r}_{\min}^j + \bar{c}_{\min}^j, \quad \bar{c}_{\min}^j \geq 0 \]

for the upper bound:

\[ \bar{r}_{\max}^j = \bar{r}_{\max}^j + \bar{c}_{\max}^j, \quad \bar{c}_{\max}^j \geq 0 \]

- The values of the shift can be assessed and changed using the values predicted by the controller for a given output

- The nonlinear steady–state plant model is used
Using output prediction to shift the constraint

Fig. 2. Idea of constraint shift calculation using output prediction generated by the predictive algorithm; the case of lower constraint
Control plant (evaporator system*)

Output Variables

$L2$ – separator level,
$X2$ – product composition,
$P2$ – operating pressure

Manipulated variables

$F2$ – product flowrate,
$P100$ – steam pressure,
$F200$ – cooling water flowrate

Fig. 3. Evaporator system

The MPCEO algorithm

• Based on the DMC type predictive algorithm,

• The manipulated variables are: steam pressure $P_{100}$ and cooling water flow $F_{200}$

• The controlled variables are: product composition $X_{2}$ and pressure in the evaporator $P_{2}$

• Measured disturbance $F_{1}$ (feed flow)

$$F_{1} = F_{10} + F_{1a} \cdot \sin(2\pi t / T_{o})$$

$F_{10} = 10$ kg/min, $F_{1a} = 0.4$ kg/min, $T_{o} = 400$ min

• The step responses obtained from environs of an operating point $P_{20} = 50.5$ kPa, $X_{20} = 25\%$
The MPCEO algorithm

• Economic performance index (cost of production)
  \[ J_E = c_1 \cdot \bar{P}100 - c_2 \cdot \bar{F}2 \]

• Constraints put on manipulated variables:
  \[ P100 \leq 400 \text{ kPa}, \; F200 \leq 400 \text{ kg/min}, \]

• The product should fulfill purity criteria:
  \[ 25 \% \leq X2 \]

• The appropriate soft constraints were put on the predicted \( X2 \) composition values

• The constraint put on \( \bar{X}2 \) set–point was as follows
  \[ \bar{X}2_{\text{min}} + \bar{r}^{X2}_{\text{min}} \leq \bar{X}2 \]
  \[ \bar{X}2_{\text{min}} = X2_{\text{min}} = 25\%, \; \bar{r}^{X2}_{\text{min}} = 0.5\% \]
Fig. 4. Responses of the control system to the step change of $F_1$ disturbance in the 170$^{th}$ minute; $X_2$ sensor fault: not taken into consideration at all, taken into consideration, additionally the constraint was shifted; failure of the $X_2$ sensor occurred in the 150th minute of simulation; above: output signals $X_2$ and $P_2$, below: control signals $P_{100}$ and $F_{200}$
Fig. 5. Responses of the control system after $X_2$ sensor fault: not taken into consideration at all, taken into consideration, additionally the constraint put on $X_2$ set–point was shifted; failure of the $X_2$ sensor occurred in the 150th minute of simulation; above: output signals $X_2$ and $P_2$, below: control signals $P_{100}$ and $F_{200}$
Fig. 6. Responses of the control system after X2 sensor fault: not taken into consideration at all, taken into consideration with: manually, dynamically changing the constraint put on X2 set-point; above: output signals X2 and P2, below: control signals P100 and F200

$J_E = 0.3254$, $J_E = -0.3171$, $J_E = -0.3471$
Summary

- Effective and relatively little complicated methods of sensor fault tolerance in control systems with predictive controllers integrated with economic optimization and output constraints were discussed.

- The methods consist in modification of the constraints taken into consideration by the algorithm.

- The methods can be used in the MPCEO algorithm with either linear or nonlinear dynamic control plant model.

- Despite simplicity of the proposed mechanisms they can offer good results thanks to the usage of both models the MPCEO algorithm is based on to improve the control system operation.