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Predictive controllers integrated with economic optimization in constrained control systems tolerating sensor faults

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Plan of presentation

1. Introduction
2. Control system
   2.1. Predictive algorithms
   2.2. Economic optimization problem
   2.3. Predictive algorithms integrated with economic optimization
3. Sensor faults handling
   3.1. Standard approach
   3.2. The case of output constraints
4. Example control system
5. Simulation experiments
6. Summary

Introduction – predictive algorithms integrated with economic optimization

- One of the methods to cope with disturbances changing quickly comparing to the dynamics of the control plant
- The steady–state control plant model is linearized
- The control system structure is simplified
- Only one quadratic optimization problem must be solved at each iteration
- The economic optimization is performed more often than in the classic hierarchical approach

Introduction – fault tolerance

- Continuation of the control system operation till the failure is fixed
- The loss of measurement means the interruption of the feedback loop
  - unstable operating point: guide the process to the region of safe operation
  - stable operating point: continue operation in the acceptable way
- Output constraints are often important for process safety and its economic effectiveness

The idea of the predictive control

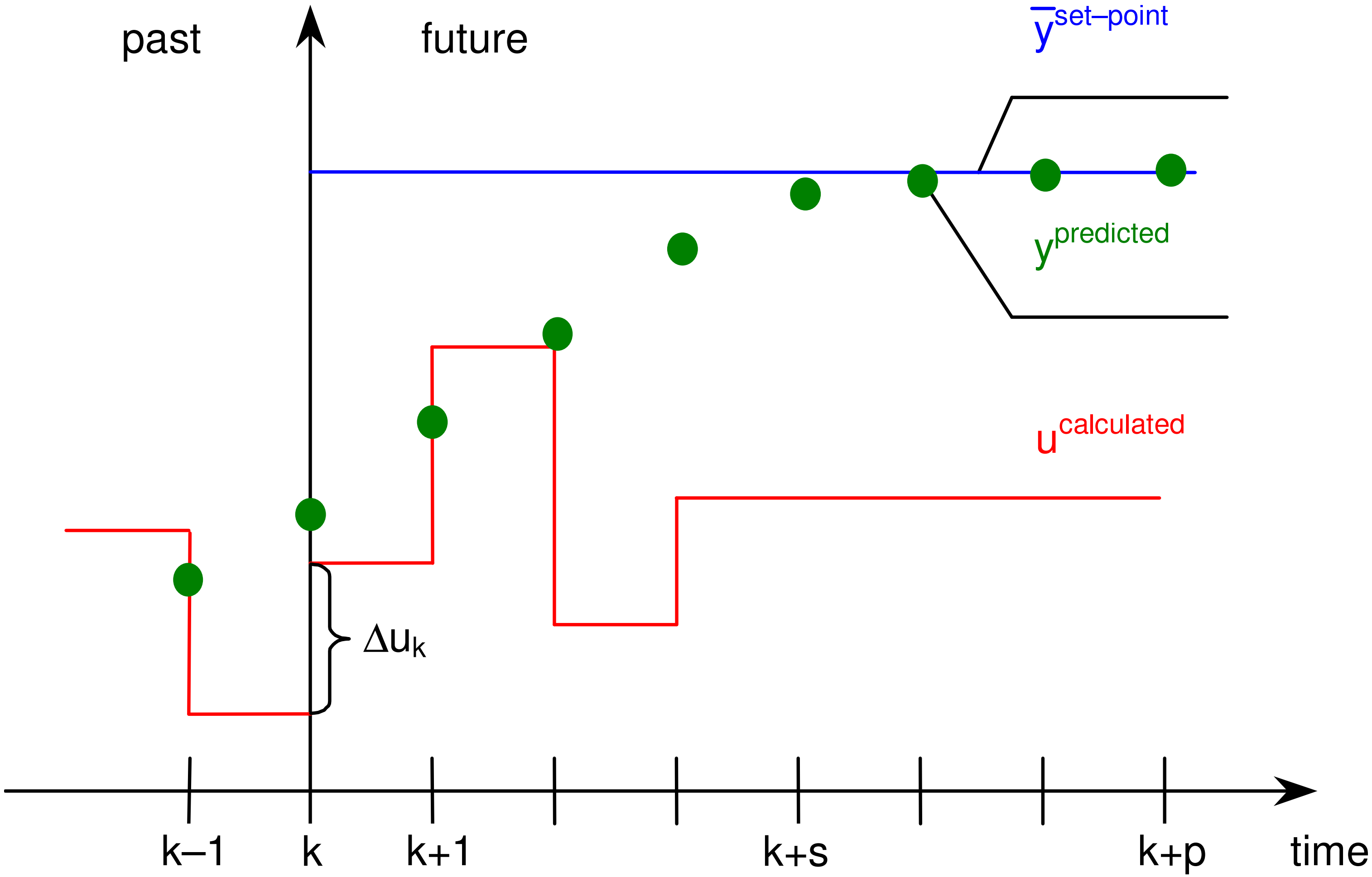

Fig. 1. Idea of predictive control; p – prediction horizon, s – control horizon, Δu_k – control signal change at current iteration

Numerical predictive control algorithms

Following problem is solved at each iteration:

$$\min_{\Delta \boldsymbol{u}} \left\{ J_{MPC} = \sum_{j=1}^{n_y} \sum_{i=1}^{p} \kappa_j \cdot \left(\overline{y}_k^j - y_{k+i|k}^j \right)^2 + \sum_{j=1}^{n_u} \sum_{i=0}^{s-1} \lambda_j \cdot \left(\Delta u_{k+i|k}^j \right)^2 \right\}$$

subject to the constraints:

$$\Delta \boldsymbol{u}_{min} \le \Delta \boldsymbol{u} \le \Delta \boldsymbol{u}_{max},$$

$$\boldsymbol{u}_{min} \le \boldsymbol{u} \le \boldsymbol{u}_{max},$$

$$\boldsymbol{y}_{min} \le \boldsymbol{y} \le \boldsymbol{y}_{max},$$

Numerical predictive control algorithms

Following problem is solved at each iteration:

$$\min_{\Delta \boldsymbol{u}} \left\{ J_{MPC} = \left(\bar{\boldsymbol{y}} - \boldsymbol{y}\right)^T \cdot \boldsymbol{\kappa} \cdot \left(\bar{\boldsymbol{y}} - \boldsymbol{y}\right) + \Delta \boldsymbol{u}^T \cdot \boldsymbol{\lambda} \cdot \Delta \boldsymbol{u} \right\}$$

subject to the constraints:

$$\Delta \boldsymbol{u}_{min} \leq \Delta \boldsymbol{u} \leq \Delta \boldsymbol{u}_{max}, \boldsymbol{u}_{min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{max}, \boldsymbol{y}_{min} \leq \boldsymbol{y} \leq \boldsymbol{y}_{max},$$

- In a nonlinear case, in order to avoid problems connected with general nonlinear optimization, effective algorithms with model linearization and quadratic optimization are used
- A few such algorithms are available, so the algorithm most suitable for a given nonlinear plant can be selected and a compromise between control performance and computation demand can be achieved

Economic optimization problem

$$\min_{\bar{\boldsymbol{y}}} J_E(\bar{\boldsymbol{y}}, \bar{\boldsymbol{u}})$$

subject to

$$\bar{\boldsymbol{u}}_{\min} \leq \bar{\boldsymbol{u}} \leq \bar{\boldsymbol{u}}_{\max}$$

$$\bar{\boldsymbol{y}}_{\min} + \bar{\boldsymbol{r}}_{\min} \leq \bar{\boldsymbol{y}} \leq \bar{\boldsymbol{y}}_{\max} - \bar{\boldsymbol{r}}_{\max}$$

$$\bar{\boldsymbol{y}} = F(\bar{\boldsymbol{u}}, \tilde{\boldsymbol{w}})$$

$F : \Re^{n_u} \times \Re^{n_w} \rightarrow \Re^{n_y}$ is a steady–state plant model (u – inputs, y – outputs, w – disturbances)

- Precise nonlinear steady–state plant model

Predictive algorithms integrated with economic optimization

$$\min_{\Delta u, \bar{y}} J_{MPC}(\bar{\boldsymbol{y}}, \Delta \boldsymbol{u}) + \gamma \cdot J_E(\bar{\boldsymbol{y}}, \bar{\boldsymbol{u}})$$

subject to

$$\Delta \boldsymbol{u}_{min} \le \Delta \boldsymbol{u} \le \Delta \boldsymbol{u}_{max},$$

$$\boldsymbol{u}_{min} \le \boldsymbol{u} \le \boldsymbol{u}_{max},$$

$$\boldsymbol{y}_{min} \le \boldsymbol{y} \le \boldsymbol{y}_{max},$$

$$\bar{\boldsymbol{u}}_{\min} \le \bar{\boldsymbol{u}} \le \bar{\boldsymbol{u}}_{\max}$$

$$\bar{\boldsymbol{y}}_{\min} + \bar{\boldsymbol{r}}_{\min} \le \bar{\boldsymbol{y}} \le \bar{\boldsymbol{y}}_{\max} - \bar{\boldsymbol{r}}_{\max}$$

$$\bar{y} = F\left(\boldsymbol{u}(k-1), \tilde{w}\right) + \boldsymbol{H}(k)\left(\bar{\boldsymbol{u}} - \boldsymbol{u}(k-1)\right)$$

economic optimization performance function

constraints from economic optimization problem

linearization of the steady-state nonlinear model *F*

Basic approach to sensor fault accommodation

- Control the loop in which the fault occurred in the open–loop configuration (feedforward control)

  — in practice: calculation of the free response using predicted instead of measured value of the output with damaged measurement

  — the problem: the disturbances acting on the control plant will not be compensated on the output with broken measurement

- Use of the disturbance measurements is crucial

The case of constraints put on output variable values

- Change (shift) of constraints in the predictive controller

$$\boldsymbol{y}_{\min} + \boldsymbol{r}_{\min} \le \boldsymbol{y} \le \boldsymbol{y}_{\max} - \boldsymbol{r}_{\max}$$

The case of constraints put on output variable values

- Change (shift) of constraints in the predictive controller

$$\boldsymbol{y}_{\min} + \boldsymbol{r}_{\min} - \tilde{\boldsymbol{y}} \leq \boldsymbol{A} \cdot \Delta\boldsymbol{u} \leq \boldsymbol{y}_{\max} - \boldsymbol{r}_{\max} - \tilde{\boldsymbol{y}}$$

  $\tilde{\boldsymbol{y}}$ is a free response

  $\boldsymbol{A}$ is a dynamic matrix

- Problem of an empty set of admissible solutions may occur

- Mechanism of soft constraints can cause violation of the constraints

The case of constraints put on output variables

- Modification of the constraints influencing the set–point values:

$$\bar{\boldsymbol{y}}_{\min} + \bar{\boldsymbol{r}}_{\min} \leq \bar{\boldsymbol{y}} \leq \bar{\boldsymbol{y}}_{\max} - \bar{\boldsymbol{r}}_{\max}$$

  for the lower bound:

$$\bar{r}^{j}_{\min} = \bar{r}^{j}_{\min} + \bar{c}^{j}_{\min}, \; \bar{c}^{j}_{\min} \geq 0$$

  for the upper bound:

$$\bar{r}^{j}_{\max} = \bar{r}^{j}_{\max} + \bar{c}^{j}_{\max}, \; \bar{c}^{j}_{\max} \geq 0$$

- The values of the shift can be assessed and changed using the values predicted by the controller for a given output

- The nonlinear steady–state plant model is used

Using output prediction to shift the constraint

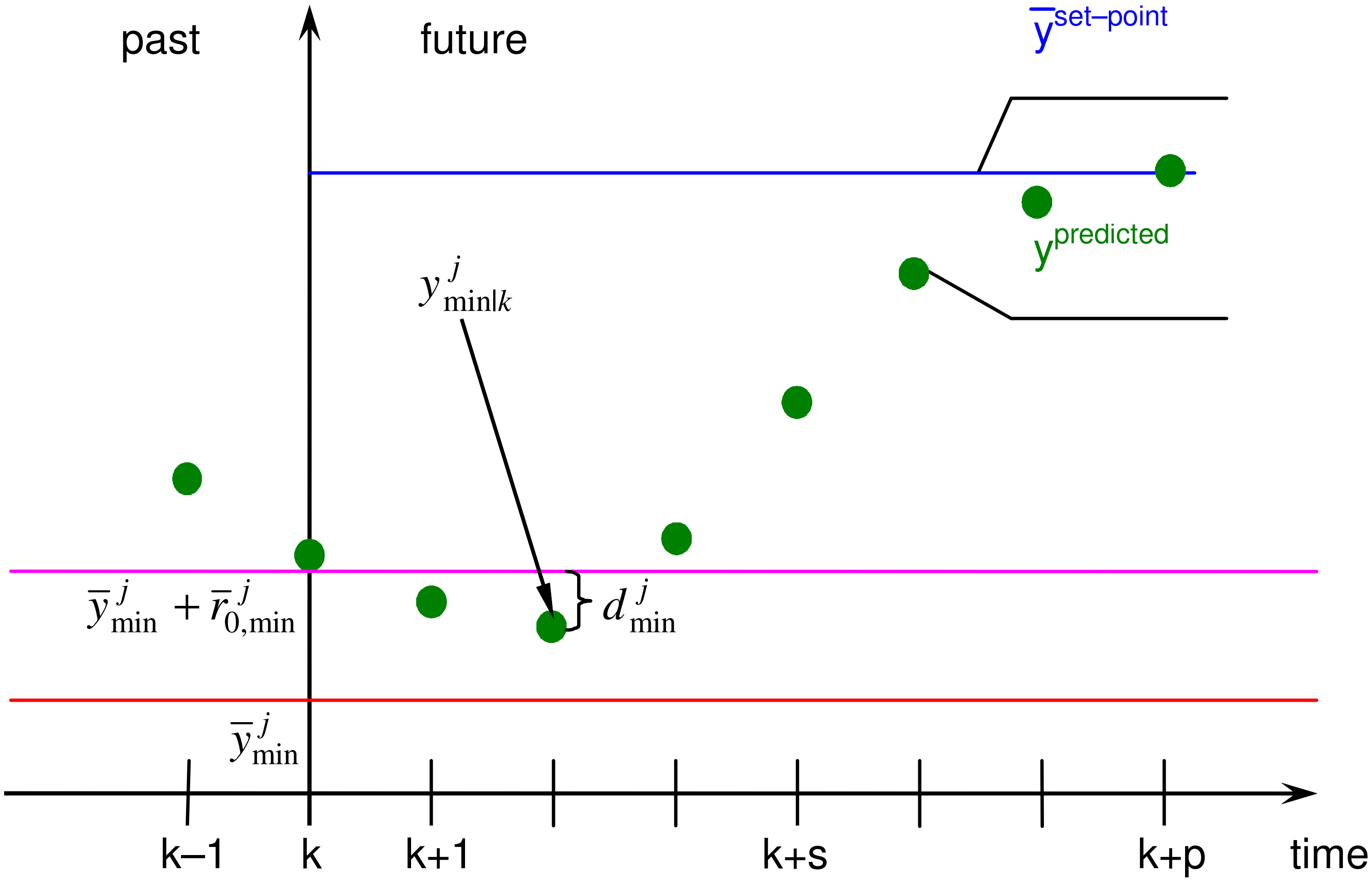

Fig. 2. Idea of constraint shift calculation using output prediction generated by the predictive algorithm; the case of lower constraint

Control plant (evaporator system)*

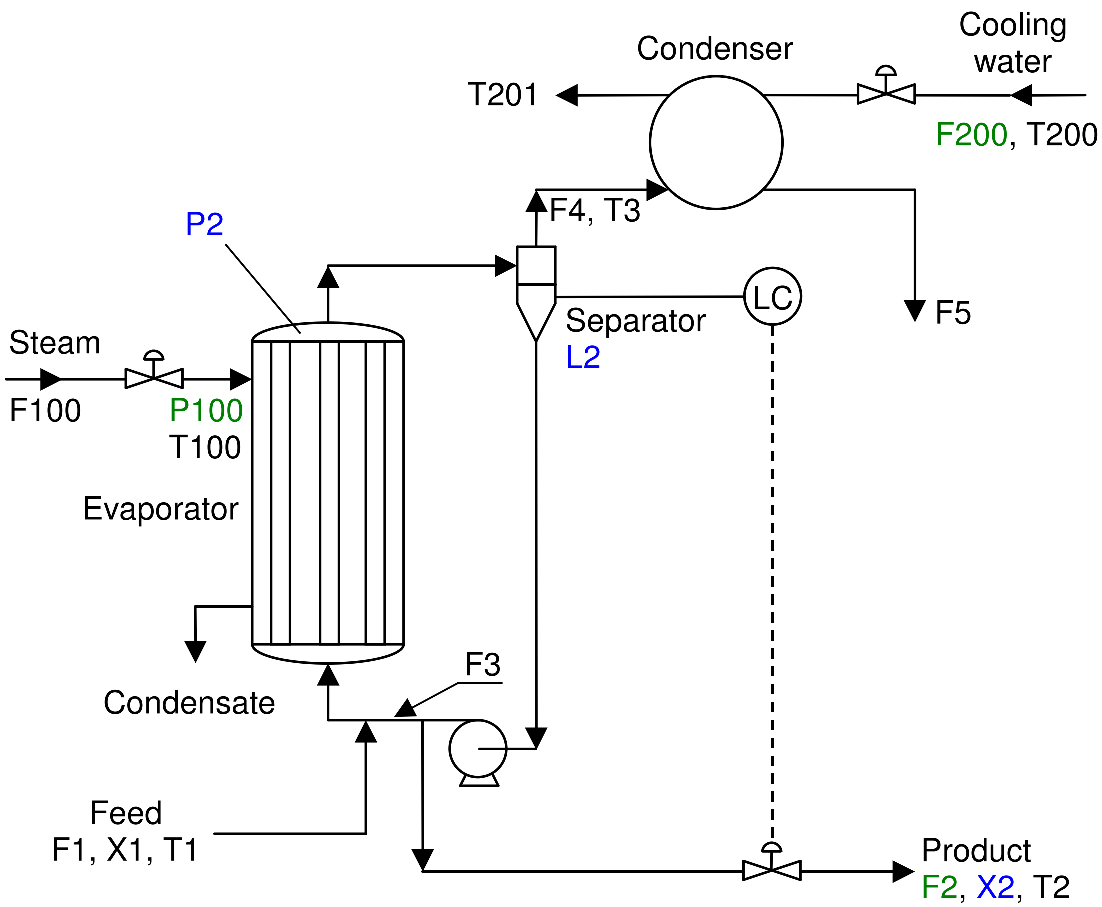

Fig. 3. Evaporator system

Output Variables
*L*2 – separator level,
*X*2 – product composition,
*P*2 – operating pressure

Manipulated variables
*F*2 – product flowrate,
*P*100 – steam pressure,
*F*200 – cooling water flowrate

* R.B. Newell, P.L. Lee: *Applied process control – a case study*; Prentice Hall, 1989

The MPCEO algorithm

- Based on the DMC type predictive algorithm,
- The manipulated variables are: steam pressure $P100$ and cooling water flow $F200$
- The controlled variables are: product composition $X2$ and pressure in the evaporator $P2$
- Measured disturbance $F1$ (feed flow)

$$F1 = F10 + F1a \cdot \sin(2 \cdot \pi \cdot t/To),$$

$F10 = 10$ kg/min, $F1a = 0.4$ kg/min, $To = 400$ min

- The step responses obtained from environs of an operating point $P20$=50.5 kPa, $X20$=25%

The MPCEO algorithm

- Economic performance index (cost of production)

$$J_E = c_1 \cdot \overline{P}100 - c_2 \cdot \overline{F}2$$

- Constraints put on manipulated variables:

$$P100 \leq 400 \text{ kPa}, \; F200 \leq 400 \text{ kg/min},$$

- The product should fulfill purity criteria:

$$25\ \% \leq X2$$

- The appropriate soft constraints were put on the predicted $X2$ composition values

- The constraint put on $\overline{X}2$ set–point was as follows

$$\overline{X}2_{\min} + \overline{r}_{\min}^{X2} \leq \overline{X}2$$

$$\overline{X}2_{\min} = X2_{\min} = 25\%, \; \overline{r}_{\min}^{X2} = 0.5\%$$

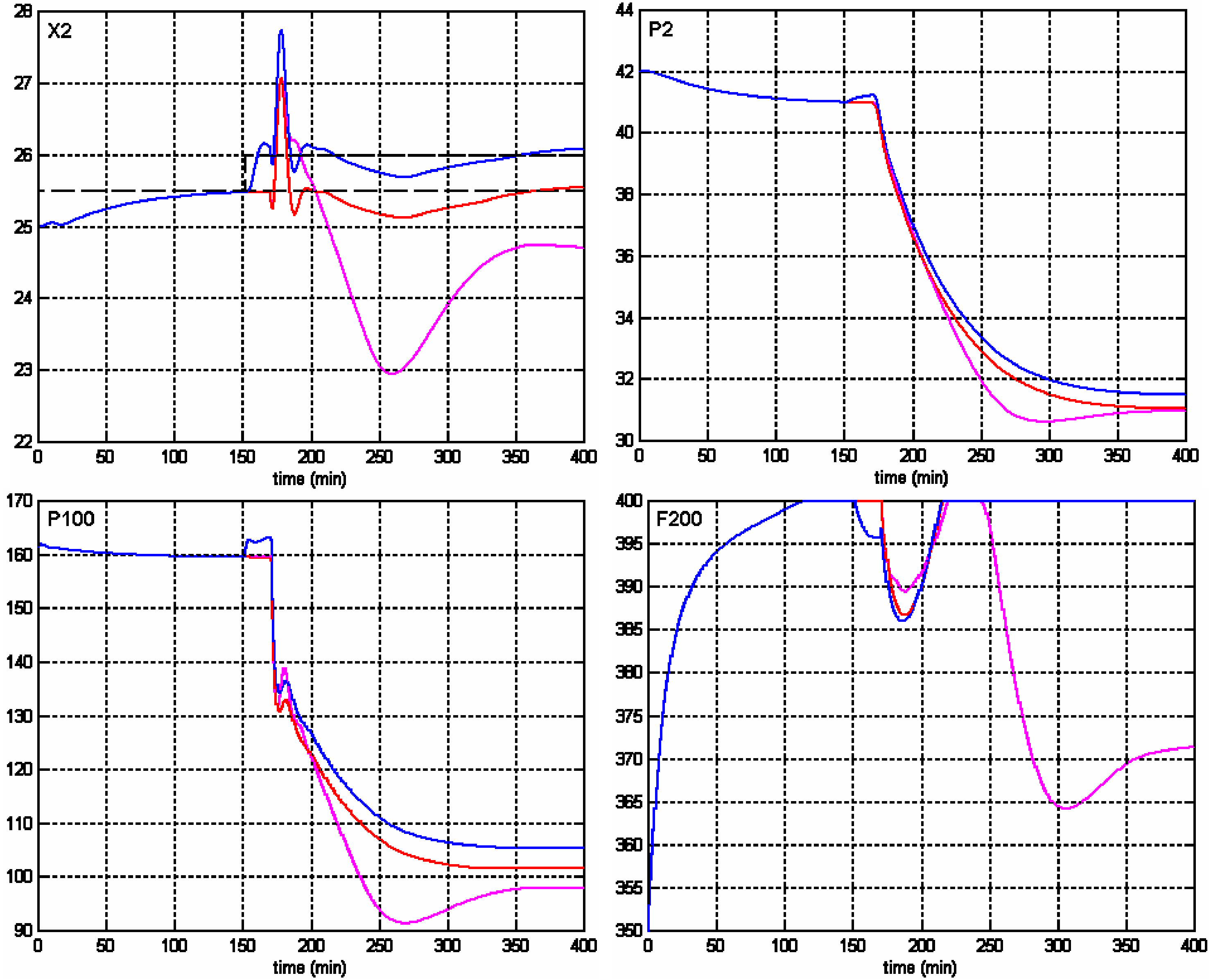

Fig. 4. Responses of the control system to the step change of $F1$ disturbance in the 170th minute; $X2$ sensor fault: not taken into consideration at all, taken into consideration, additionally the constraint was shifted; failure of the $X2$ sensor occurred in the 150th minute of simulation; above: output signals $X2$ and $P2$, below: control signals $P100$ and $F200$

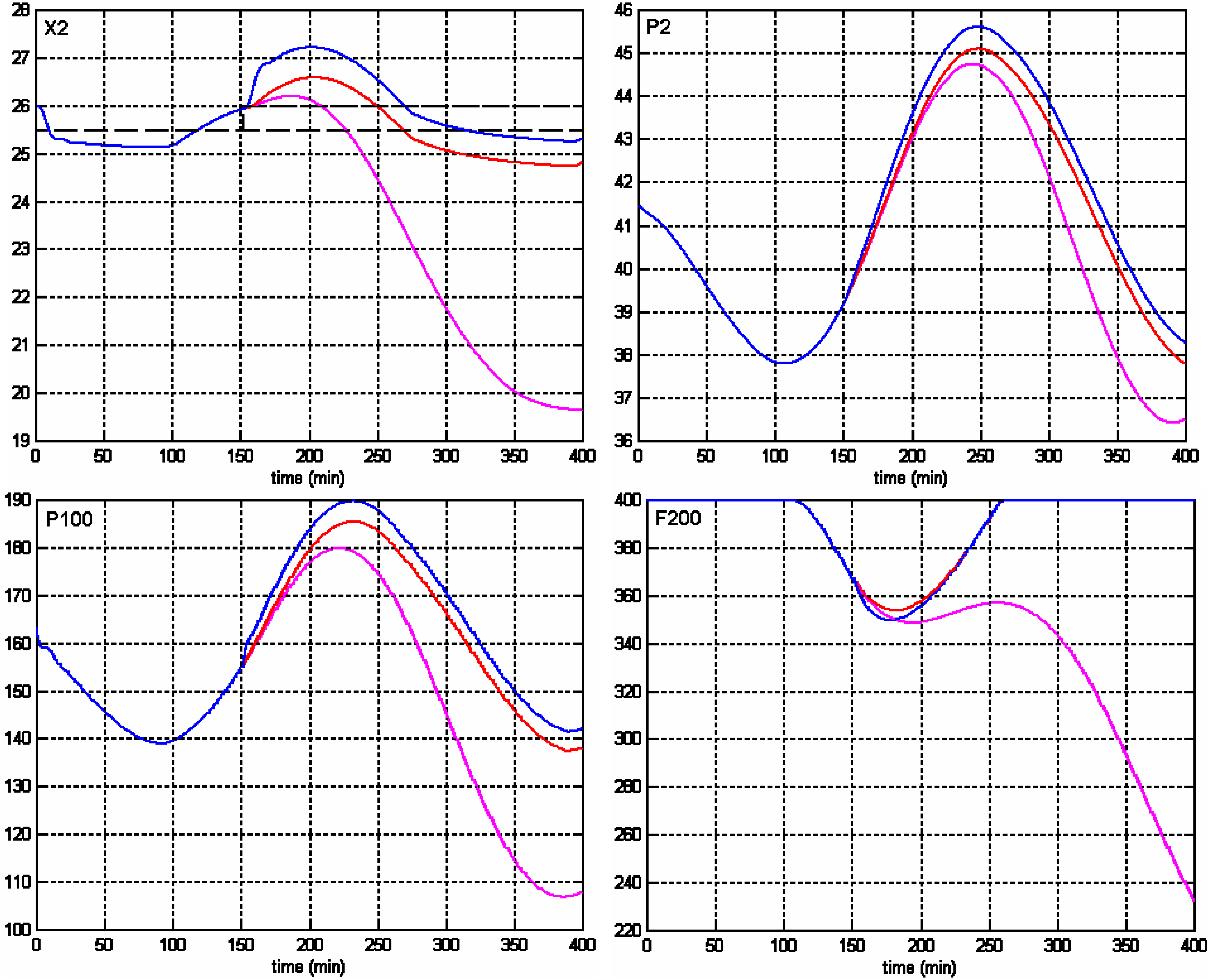

Fig. 5. Responses of the control system after $X2$ sensor fault: not taken into consideration at all, taken into consideration, additionally the constraint put on $X2$ set–point was shifted; failure of the $X2$ sensor occurred in the 150th minute of simulation; above: output signals $X2$ and $P2$, below: control signals $P100$ and $F200$

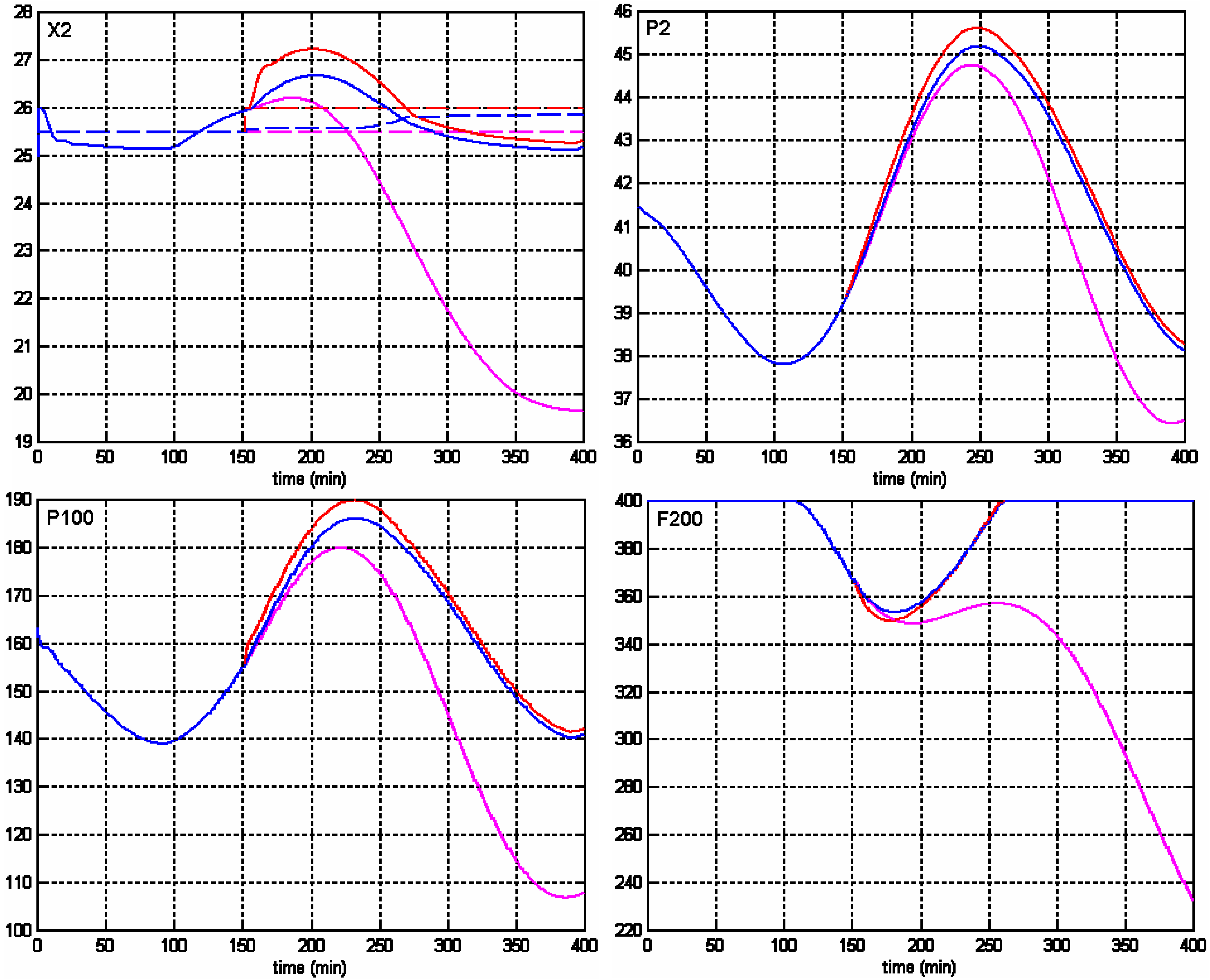

Fig. 6. Responses of the control system after $X2$ sensor fault: not taken into consideration at all, taken into consideration with: manually, dynamically changing the constraint put on $X2$ set–point; above: output signals $X2$ and $P2$, below: control signals $P100$ and $F200$

$J_E=0.3254$, $J_E=-0.3171$, $J_E=-0.3471$

Summary

- Effective and relatively little complicated methods of sensor fault toleration in control systems with predictive controllers integrated with economic optimization and output constraints were discussed
- The methods consist in modification of the constraints taken into consideration by the algorithm
- The methods can be used in the MPCEO algorithm with either linear or nonlinear dynamic control plant model
- Despite simplicity of the proposed mechanisms they can offer good results thanks to the usage of both models the MPCEO algorithm is based on to improve the control system operation