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Fuzzy Dynamic Matrix Control algorithms for nonlinear plants

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SCHEME OF PRESENTATION:

Introduction

DMC algorithm (C.R. Cutler and B.L. Remarker, 1979; C.E. Garcia, A.M. Morshedi, 1984):

- a long–range horizon predictive control algorithm;
- many advantages – possibility of taking into account: constraints, future set–point changes, anticipated disturbance changes.

Takagi–Sugeno fuzzy models (an outline):

- division of an operational space into regions in a fuzzy way;
- a linear model in each region;
- the output of the whole, fuzzy model is composed of the outputs of all local models (a soft switching between regions is assured).

Linear DMC algorithm based on analytical formulation

The basic idea of the DMC algorithm:

$$\min_{\Delta u} \sum_{i=1}^{p} \left(y_k^{sp} - y_{k+i}^{pred} \right)^2 + \lambda \cdot \sum_{i=0}^{s-1} \left(\Delta u_{k+i} \right)^2$$

$$y_{k+i}^{pred} = y_k + w_{k+i} + \Delta y_{k+i} \,,\; i = 1,\ldots, p,$$

A unique solution:

$$\boldsymbol{\Delta u} = (\boldsymbol{A}^T \cdot \boldsymbol{A} + \lambda \cdot \boldsymbol{I})^{-1} \cdot \boldsymbol{A}^T \cdot (\boldsymbol{e} - \boldsymbol{w}),$$

where

$$\Delta u = \left[\Delta u_k, \ldots, \Delta u_{k+s-1} \right]^T \text{ – future control increments,}$$

$$\Delta y = \left[\Delta y_{k+1}, \ldots, \Delta y_{k+p} \right]^T \text{ – future output increments,}$$

$$w = \left[w_{k+1}, \ldots, w_{k+p} \right]^T \text{ – depends on the past,}$$

$$e = \left[y_k^{sp} - y_k, \ldots, y_k^{sp} - y_k \right]^T$$

Only the first element of $\boldsymbol{\Delta u}$ is used at each time step.

The structure of the controller:

$$u_k = u_{k-1} + r_0 \cdot e_k + \sum_{j=1}^{l} r_j \cdot \Delta u_{k-j}$$

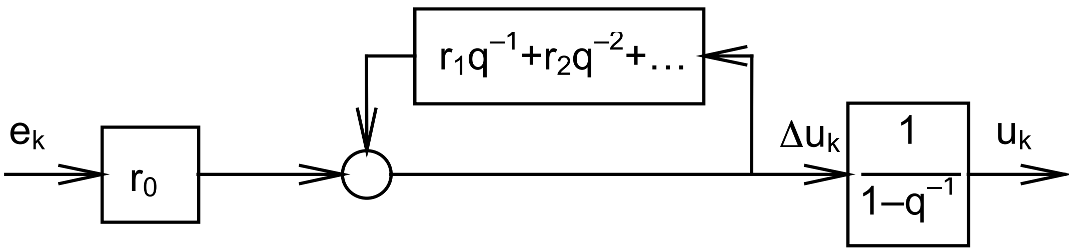

Fig. 1. Block diagram of the DMC controller

Fuzzy DMC algorithms based on analytical formulation

FDMC 1

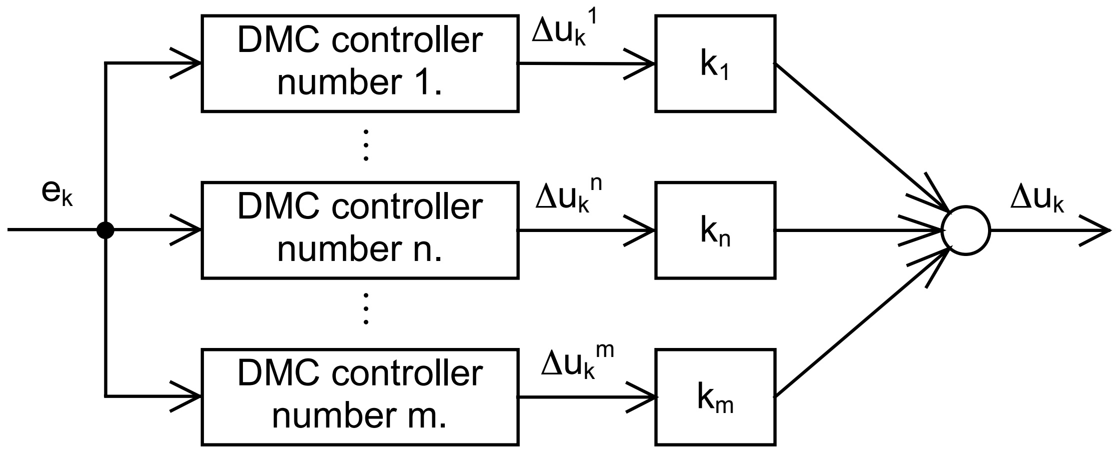

Fig. 2. Block diagram of the FDMC1 controller; Δu_k^n – outputs of sub–controllers, k_n– weights, (n = 1,…, m), m – number of regions

FDMC 2

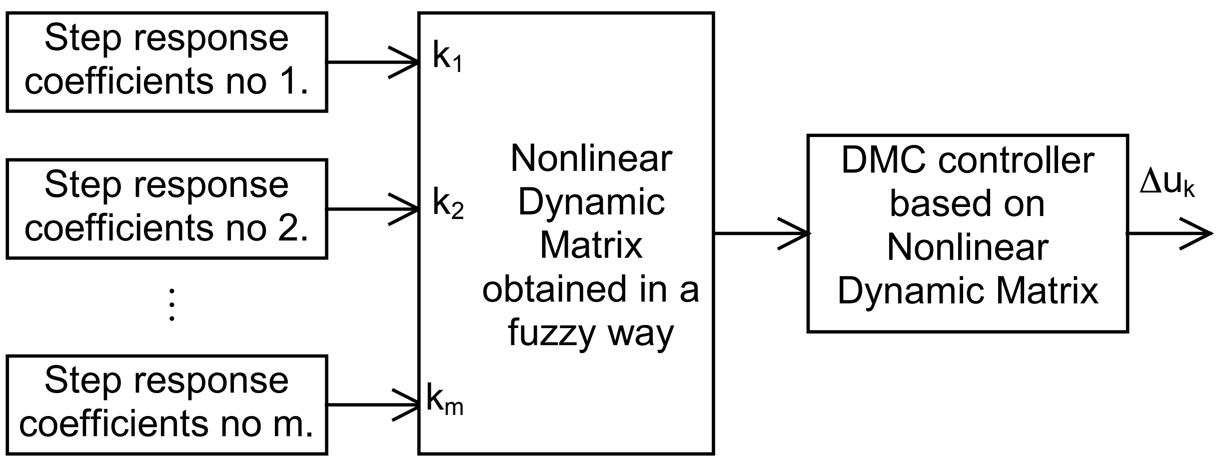

Fig. 3. FDMC2 algorithm structure

Linear DMC algorithm based on numerical formulation

The optimization problem solved at each time step:

$$\min_{\Delta u} \sum_{i=1}^{p} \left(y_{k+i}^{sp} - y_{k+i}^{pred} \right)^2 + \lambda \cdot \sum_{i=0}^{s-1} \left(\Delta u_{k+i} \right)^2 ,$$

subject to the constraints:

$$\boldsymbol{\Delta u_{min}} \leq \boldsymbol{\Delta u} \leq \boldsymbol{\Delta u_{max}},$$

$$\boldsymbol{u_{min}} \leq \boldsymbol{u} \leq \boldsymbol{u_{max}},$$

$$\boldsymbol{y_{min}} \leq \boldsymbol{y} \leq \boldsymbol{y_{max}},$$

where

$$\Delta u = \left[\Delta u_k, \ldots, \Delta u_{k+s-1} \right]^T ,$$

$$u = \left[u_{k-1} + \Delta u_k, \ldots, u_{k-1} + \sum_{i=0}^{s-1} \Delta u_{k+i} \right]^T ,$$

$$y = \left[y_{k+1}^{pred}, \ldots, y_{k+p}^{pred} \right]^T .$$

Fuzzy DMC algorithms based on numerical formulation

FDMC 3

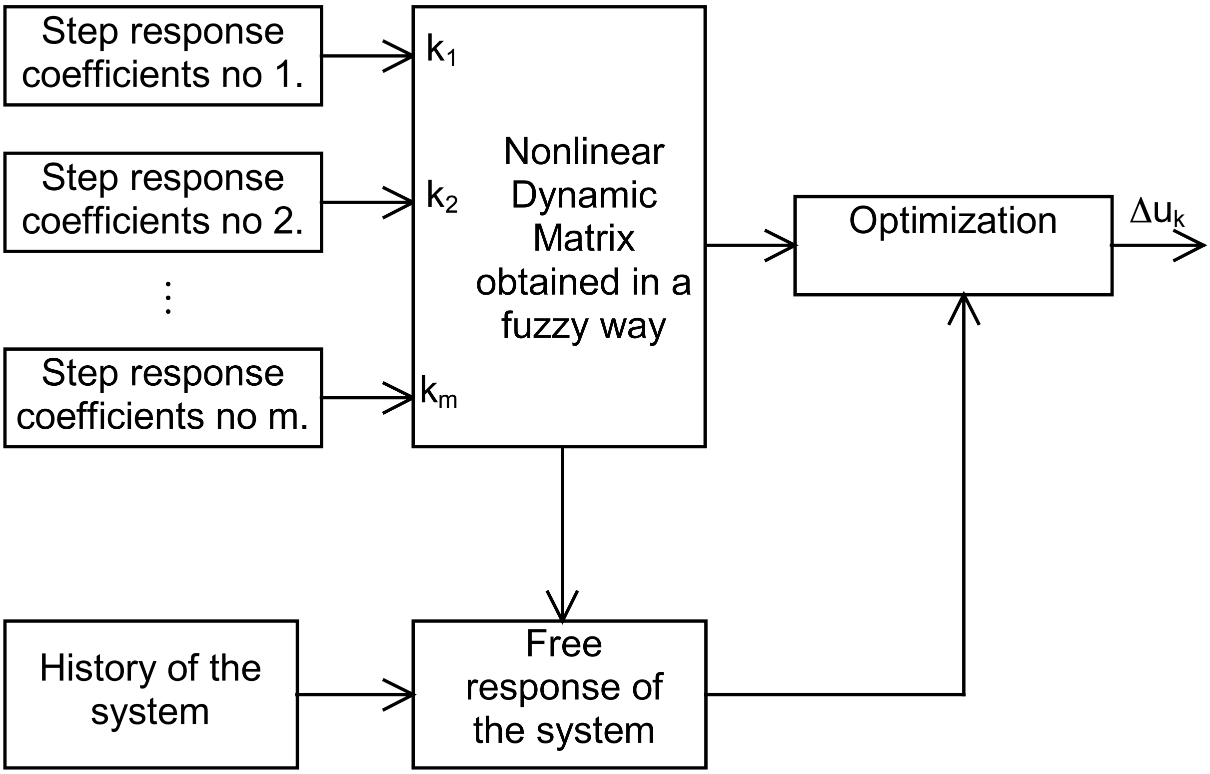

Fig. 4. FDMC3 algorithm structure

FDMC 4

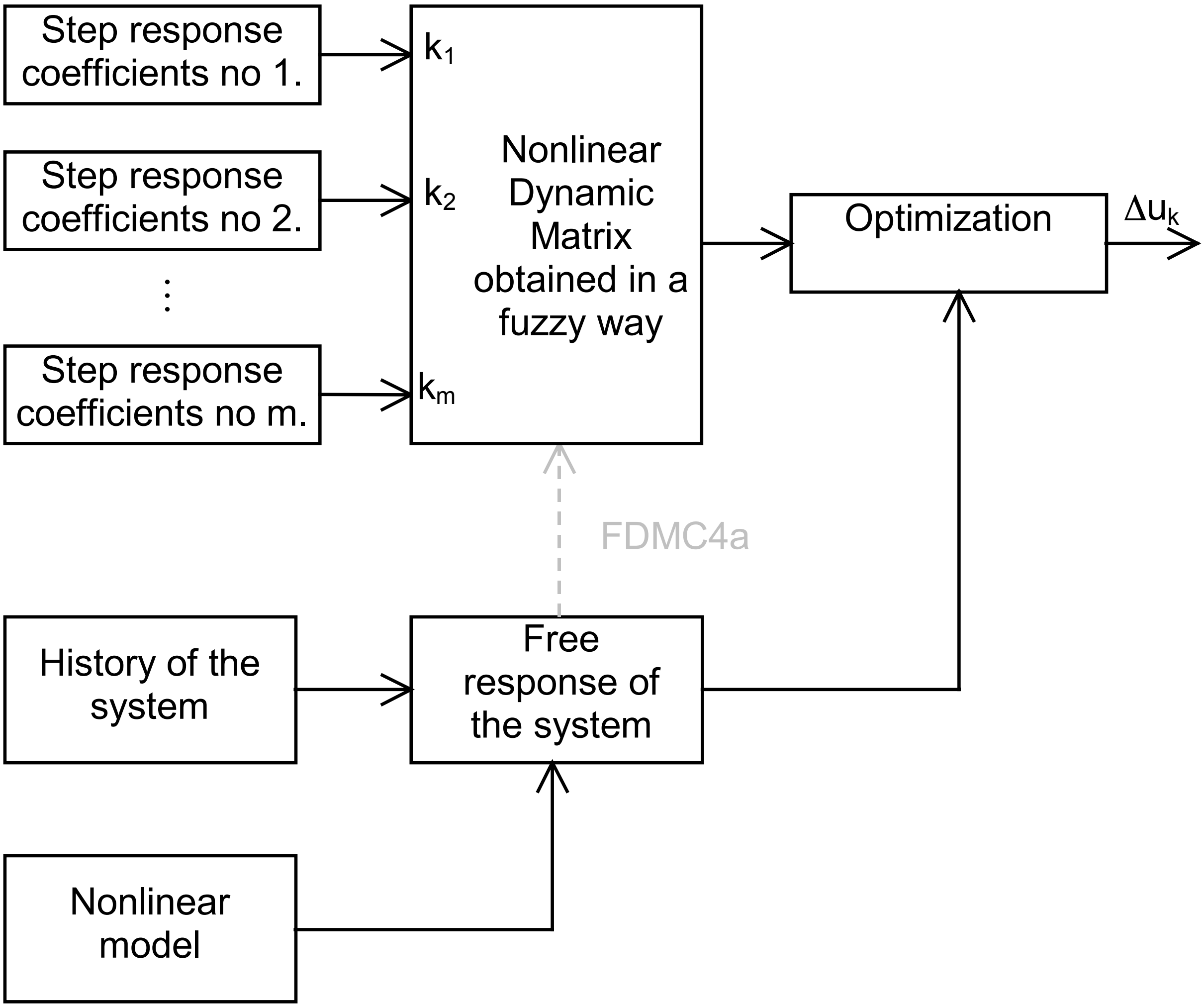

Fig. 5. FDMC4, FDMC4a algorithm structure

Control plant

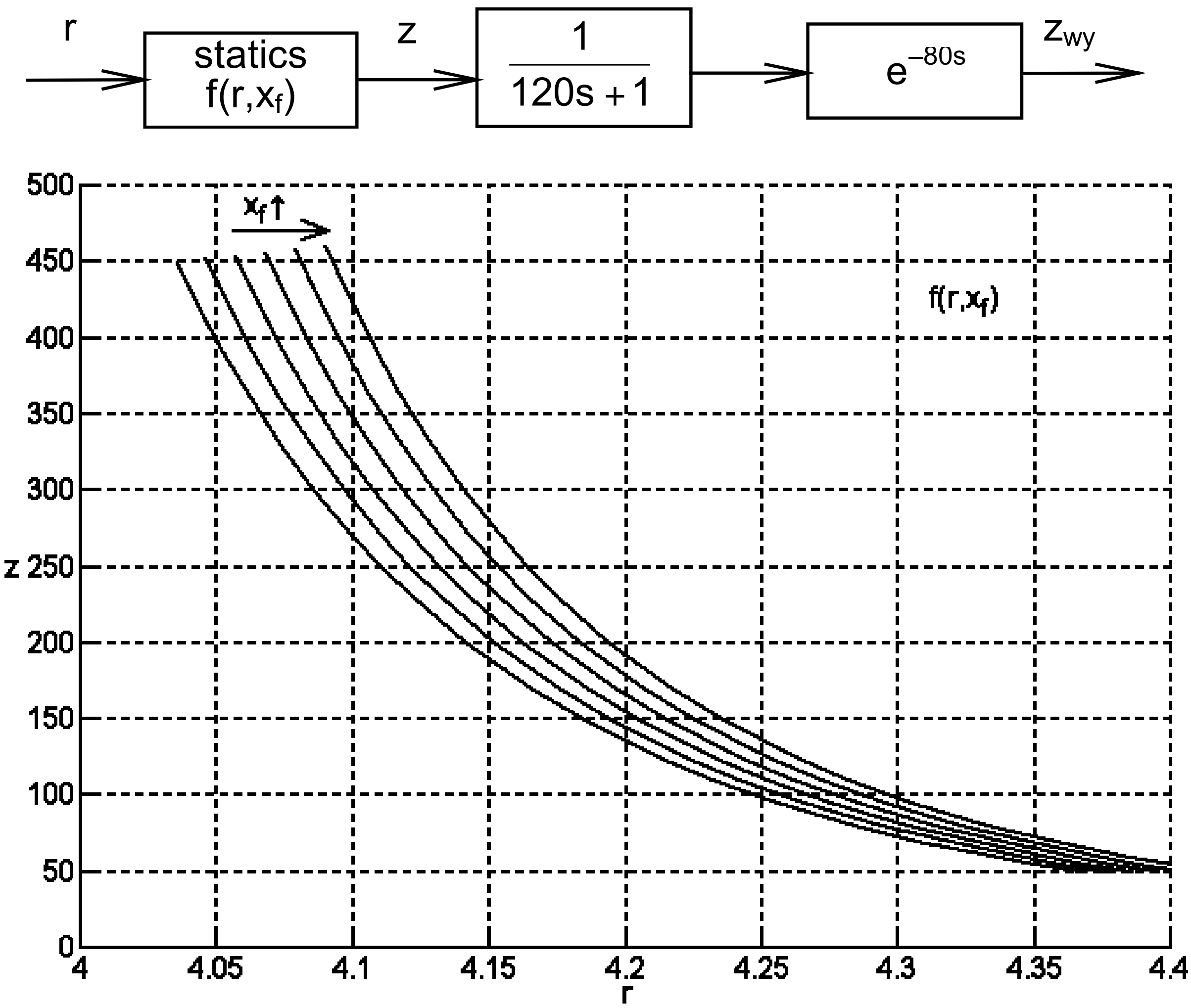

Fig. 6. Block diagram and static characteristics of the control plant

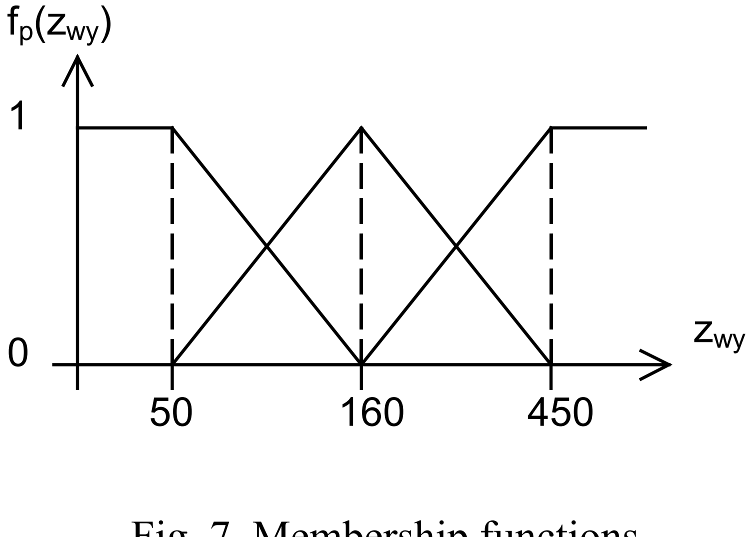

Fig. 7. Membership functions

a)
b)

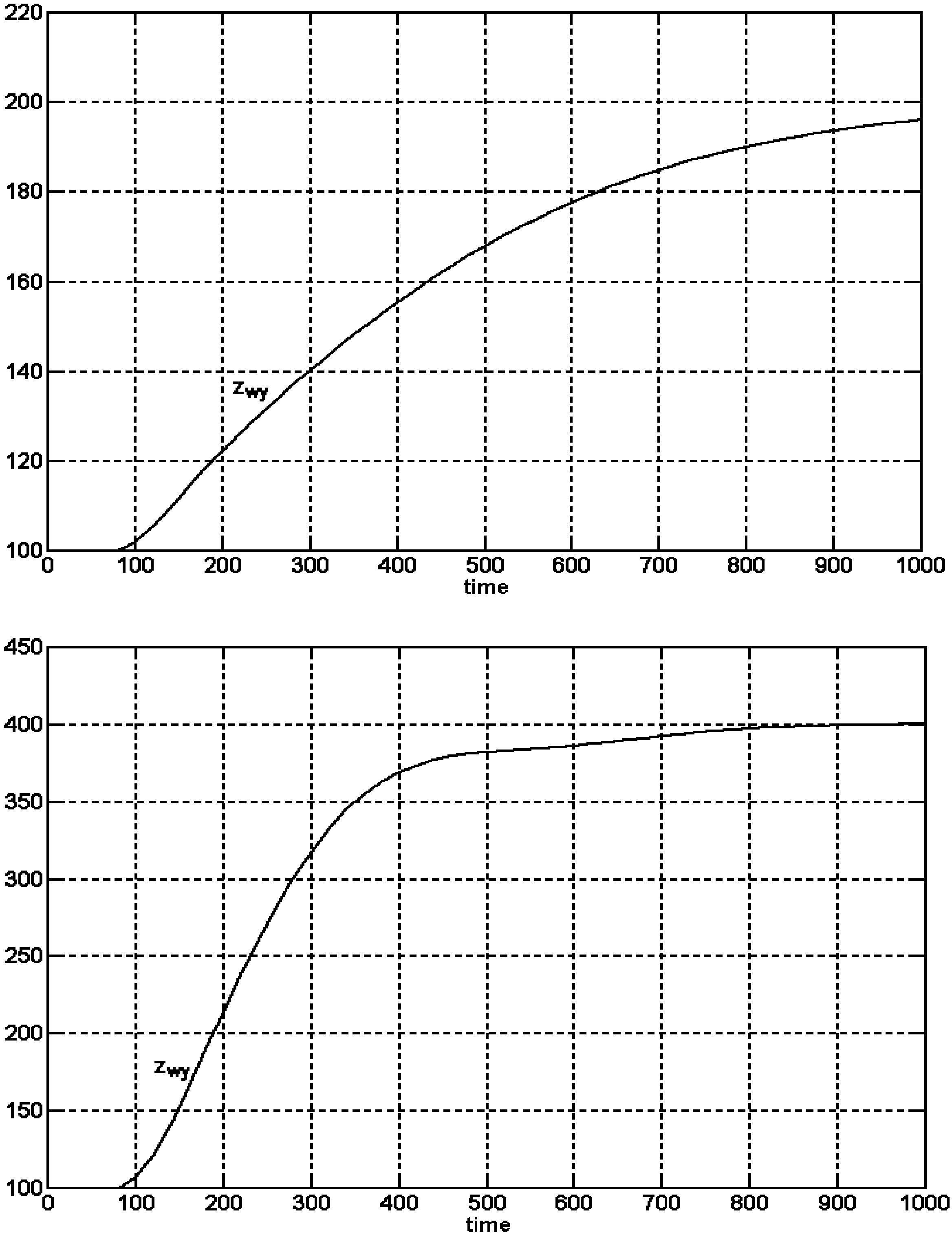

Fig. 8. Responses with "normal" DMC controller designed for set–point 400 ppm; set–point change from z_0 = 100 ppm to a) z_{zad} = 200 ppm; b) z_{zad} = 400 ppm;

a)

b)

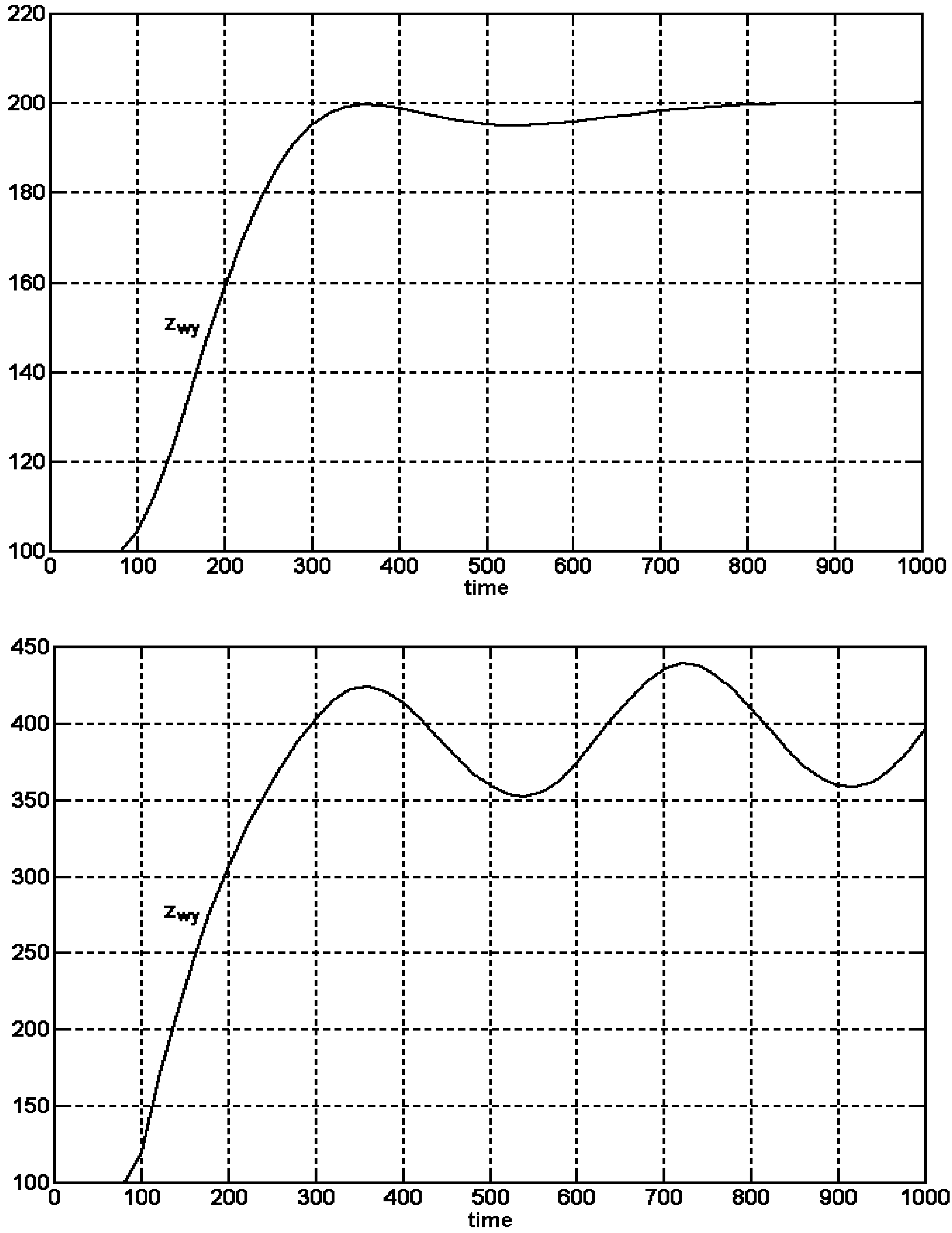

Fig. 9. Responses with "normal" DMC controller designed for set–point 200 ppm; set–point change from z_0 = 100 ppm to a) z_{zad} = 200 ppm; b) z_{zad} = 400 ppm;

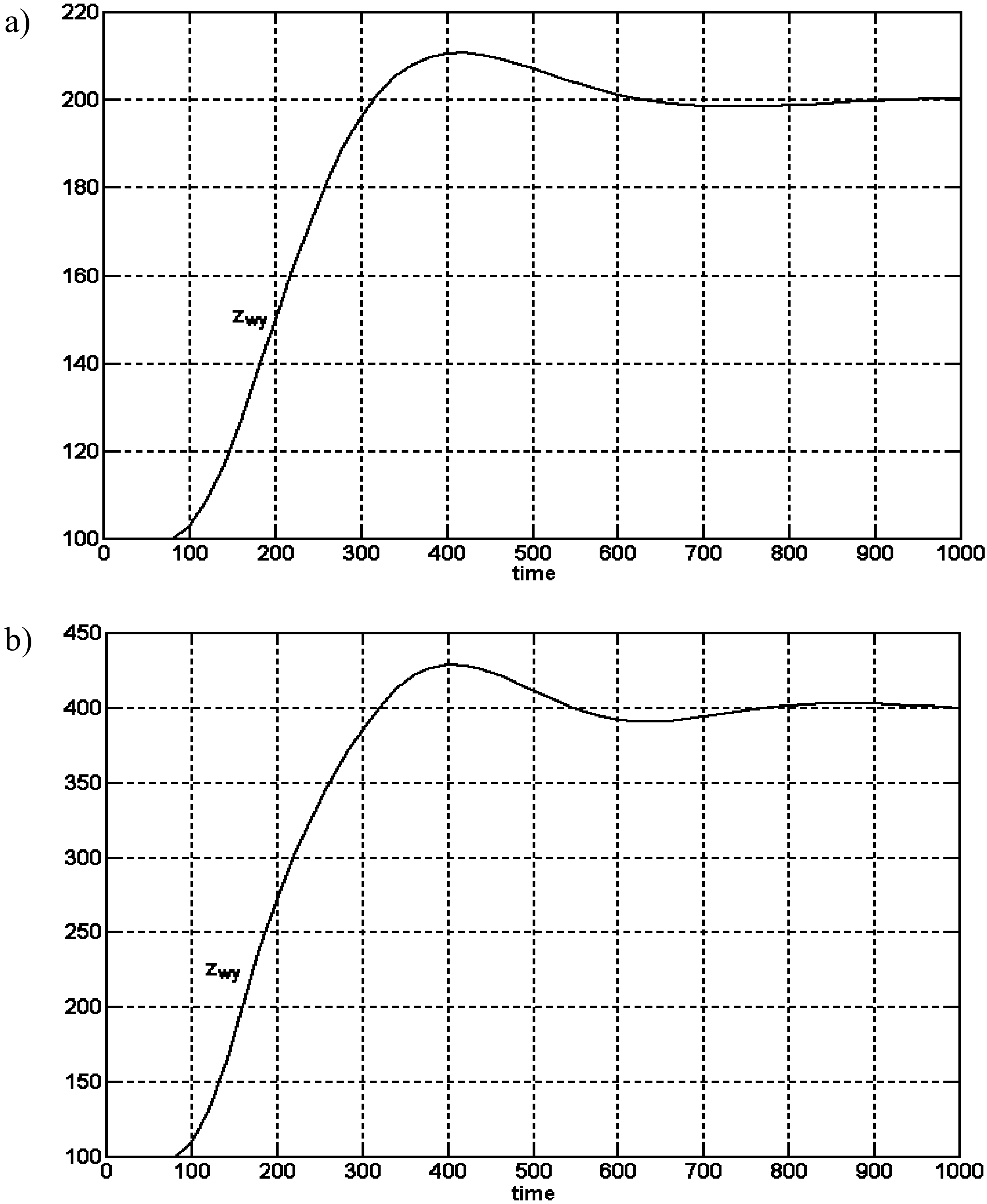

Fig. 10. Responses with fuzzy DMC controller;
set–point change from z_0 = 100 ppm to
a) z_{zad} = 200 ppm; b) z_{zad} = 400 ppm;

a) b)

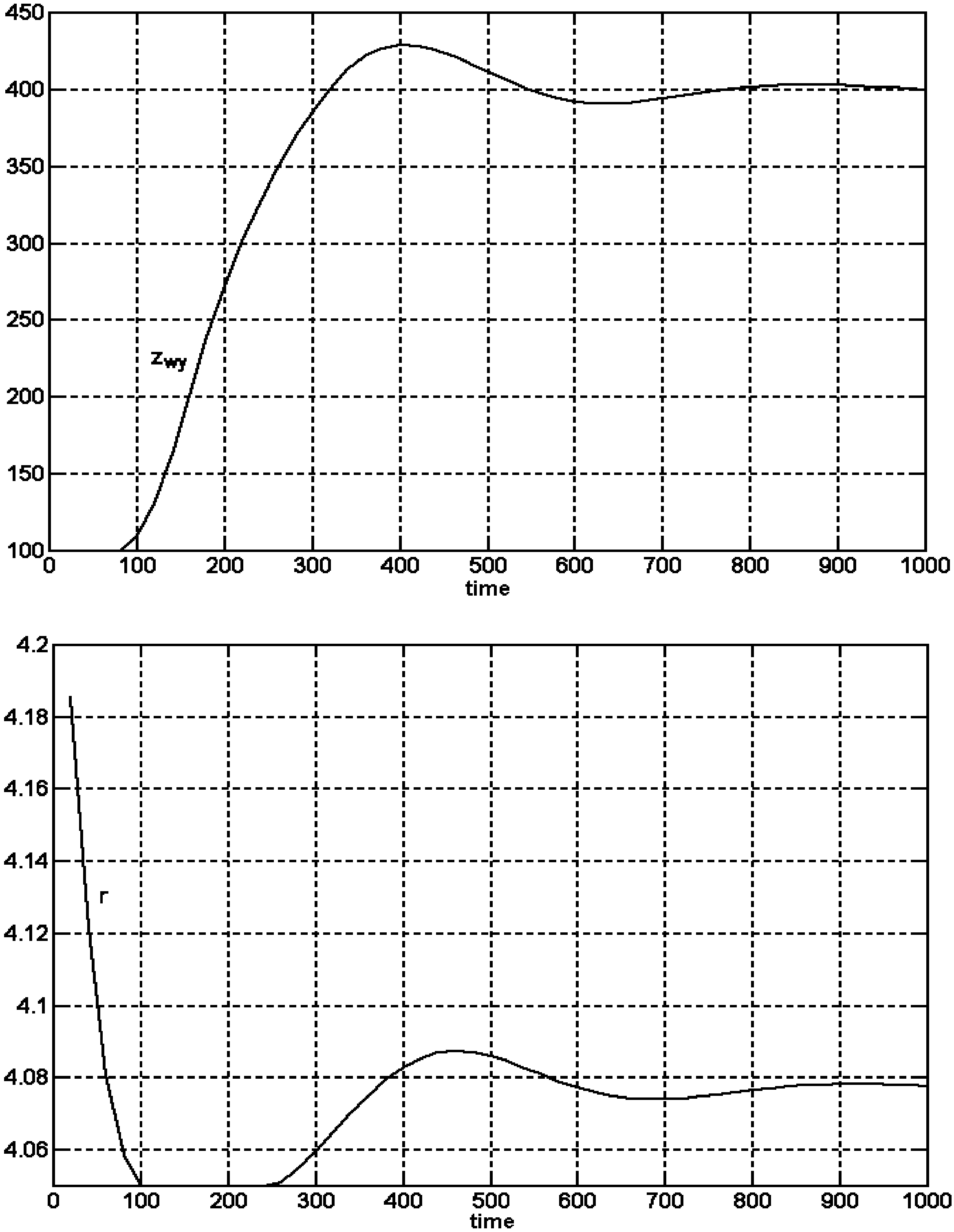

Fig. 11. Responses with fuzzy DMC controller;
set–point change from z_0 = 100 ppm to z_{zad} = 400 ppm;
a) output variable; b) manipulated variable;

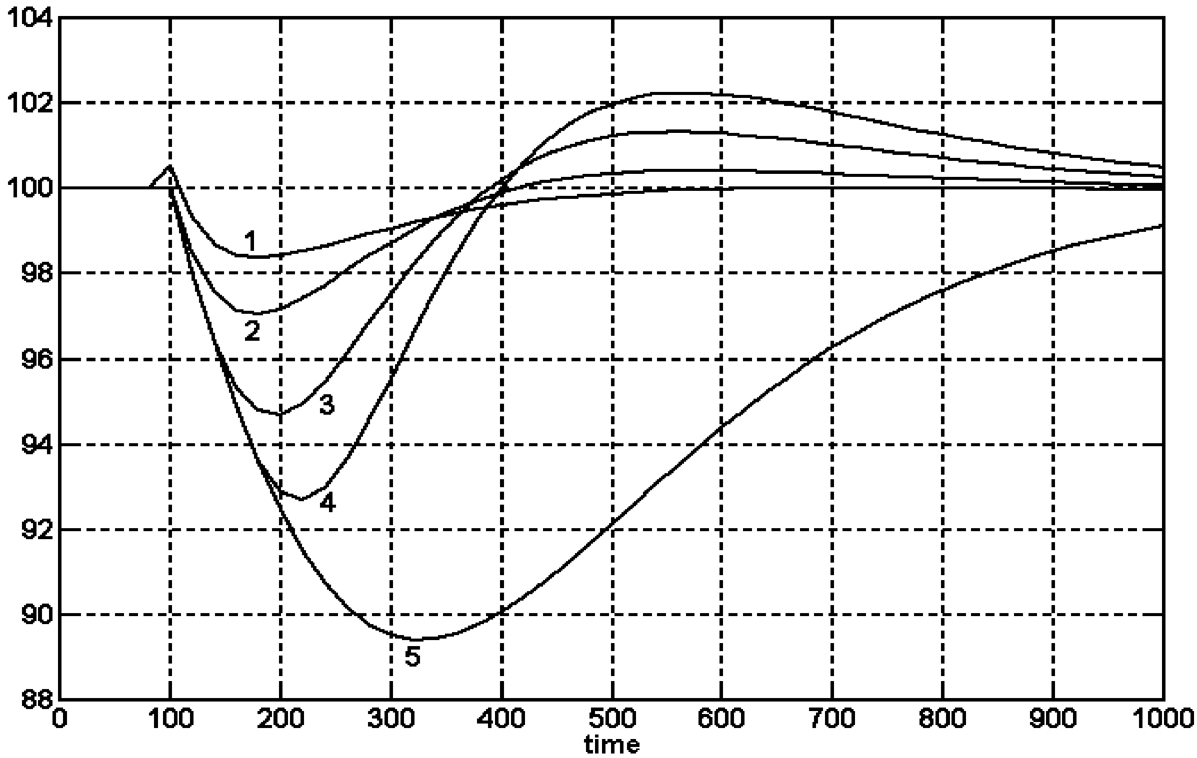

Fig. 12. Responses with fuzzy DMC controller for the step change of measurable disturbance from $x_f = 0{,}81$ to $x_{fb} = 0{,}8$;

1 – one step ahead anticipation of the disturbance change,

2 – immediate measurement,

3 – measurement with delay $2 \cdot T_p$,

4 – measurement with delay $4 \cdot T_p$,

5 – without disturbance measurement.

Summary

- Fuzzy DMC controllers are combination of long–range horizon DMC predictive controller idea with the fuzzy Takagi–Sugeno (multiregional) approach. This union enables to include the advantages of both techniques into the proposed controllers.
- There is a possibility to choose a controller version most suitable for a problem at hand. Starting from the least complicated FDMC1 through more advanced controllers solving quadratic optimization problem with constraints and using linear prediction (FDMC3), to controllers using nonlinear prediction and model adaptation at each control time step (FDMC4).
- The efficient suboptimal handling of control constraints in the analytic approach (P. Marusak, J. Pułaczewski, P. Tatjewski; 1999) can be immediately embedded into fuzzy DMC formulations.