Predictive controllers integrated with economic optimization tolerating actuator faults: application to a nonlinear plant

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**Introduction**

- **Algorithms**: Model Predictive Control algorithms integrated with Economic Optimization (MPCEO)
- **Aim**: Continuation of the control system operation till the failure is fixed
- **Assumption**: Methods of detection and isolation of actuator faults are available, measurement of the actuator output, in particular
- **Remark**: Output constraints are often important for safety and economic effectiveness of the process
The idea of the predictive control

Fig. 1. Idea of predictive control; \( p \) – prediction horizon, \( s \) – control horizon, \( \Delta u_k \) – control signal change at current iteration
Numerical predictive control algorithms

Following problem is solved at each iteration:

$$\min_{\Delta u} \left\{ J_{MPC} = \sum_{j=1}^{n_y} \sum_{i=1}^{p} \kappa_j \cdot (\bar{y}_k^j - y_{k+il}^j)^2 + \sum_{j=1}^{n_u} \sum_{i=0}^{s-1} \lambda_j \cdot (\Delta u_{k+il}^j)^2 \right\}$$

subject to the constraints:

$$\Delta u_{\text{min}} \leq \Delta u \leq \Delta u_{\text{max}},$$

$$u_{\text{min}} \leq u \leq u_{\text{max}},$$

$$y_{\text{min}} \leq y \leq y_{\text{max}},$$
Numerical predictive control algorithms

Following problem is solved at each iteration:

$$\min_{\Delta u} \left\{ J_{MPC} = \sum_{j=1}^{n_y} \sum_{i=1}^{p} \kappa_j \cdot \left( \overline{y}_j^k - y_{j+k}^k \right)^2 + \sum_{j=1}^{n_u} \sum_{i=0}^{s-1} \lambda_j \cdot \left( \Delta u_{j+k}^k \right)^2 \right\}$$

subject to the constraints:

$$\Delta u_{\min} \leq \Delta u \leq \Delta u_{\max}, \ u_{\min} \leq u \leq u_{\max}, \ y_{\min} \leq y \leq y_{\max},$$

• In a nonlinear case, in order to avoid problems connected with general nonlinear optimization, effective algorithms with model linearization and quadratic optimization are used

• A few such algorithms are available, so the algorithm most suitable for a given nonlinear plant can be selected and a compromise between control performance and computation demand can be achieved
**Economic optimization problem**

\[
\min_{\bar{y}} J_E(\bar{y}, \bar{u})
\]

subject to

\[
\bar{u}_{\text{min}} \leq \bar{u} \leq \bar{u}_{\text{max}}
\]

\[
\bar{y}_{\text{min}} + \bar{r}_{\text{min}} \leq \bar{y} \leq \bar{y}_{\text{max}} - \bar{r}_{\text{max}}
\]

\[
\bar{y} = F(\bar{u}, \hat{w})
\]

\[F : \mathbb{R}^{n_u} \times \mathbb{R}^{n_w} \rightarrow \mathbb{R}^{n_y}\] is a steady–state plant model (\(u - \) inputs, \(y - \) outputs, \(w - \) disturbances)

- Precise nonlinear steady–state plant model
Classical multilayer control system structure

Fig. 2. Hierarchical control system structure with MPC advanced control layer
MPC Integrated with Economic Optimization (MPCEO)

Fig. 3. Control system structure with MPC integrated with economic optimisation
**MPC Integrated with Economic Optimization (MPCEO)**

- One of the methods to cope with disturbances changing quickly comparing to the dynamics of the control plant
- The steady–state control plant model is linearized
- Only one quadratic optimization problem must be solved at each iteration
- The control system structure is simplified
- The economic optimization is performed more often than in the classic hierarchical approach
MPC Integrated with Economic Optimization (MPCEO)

$$\min_{\Delta u, \bar{y}} J_{\text{MPC}}(\bar{y}, \Delta u) + \gamma \cdot J_E(\bar{y}, \bar{u})$$

subject to

- Constraints from economic optimization problem:
  $$\Delta u_{\text{min}} \leq \Delta u \leq \Delta u_{\text{max}},$$
  $$u_{\text{min}} \leq u \leq u_{\text{max}},$$
  $$y_{\text{min}} \leq y \leq y_{\text{max}},$$

- Economic optimization performance function:
  $$\bar{u}_{\text{min}} \leq \bar{u} \leq \bar{u}_{\text{max}}$$

- Linearization of the steady-state nonlinear model $F$:
  $$\bar{y} = F(u(k-1), \tilde{w}) + H(k)(\bar{u} - u(k-1))$$
Actuator faults handling

• A set of equality constraints is added to the algorithm after fault detection

\[ \Delta u_k^{m+il_k} = 0, i = 0, \ldots, s - 1 \]

\( m \) – number of control signal affected by the failure

• Application is relatively easy

• Elimination of some decision variables
Actuator faults handling

• Equality constraint is added

\[ \bar{u}^m = u_{bl}^m \]

\( m \) – number of control signal affected by the failure

\( u_{bl}^m \) – output of the actuator

• Easy application

• The steady–state model is in practice modified

• Measurement of the actuator output is often available
Control plant (evaporator system*)

**Output Variables**

- $L_2$ – separator level,
- $X_2$ – product composition,
- $P_2$ – operating pressure

**Manipulated variables**

- $F_2$ – product flowrate,
- $P_{100}$ – steam pressure,
- $F_{200}$ – cooling water flowrate

Fig. 4. Evaporator system

**The MPCEO algorithm**

- **The manipulated variables** are: steam pressure $P_{100}$ and cooling water flow $F_{200}$
- **The controlled variables** are: product composition $X_2$ and pressure in the evaporator $P_2$
- Measured disturbance $F_1$ (feed flow)
- Based on the fuzzy DMC predictive algorithm,
The MPCEO algorithm

Parameters:

\[ \kappa_{P_2} = \kappa_{X_2} = 1, \]
\[ \lambda_{P_{100}} = \lambda_{F_{200}} = \lambda_{F_3} = 0.1 \]
\[ p = 100, \quad s = 10 \]

Fig. 5. Membership functions of the fuzzy MPCEO controller
**The MPCEO algorithm**

- Economic performance index (cost of production)
  \[ J_E = c_1 \cdot \overline{P}_{100} - c_2 \cdot \overline{F}_2 \]

- Constraints put on manipulated variables:
  \[ 0 \text{ kPa} \leq P_{100} \leq 400 \text{ kPa}, \ 0 \text{ kg/min} \leq F_{200} \leq 400 \text{ kg/min}, \]

- The product should fulfill purity criteria:
  \[ 25 \% \leq X_2 \]

- The appropriate soft constraints were put on the predicted \( X_2 \) composition values

- The constraint put on \( \overline{X}_2 \) set–point was as follows
  \[ \overline{X}_{2\text{\_min}} + \overline{r}_{X_2} \leq \overline{X}_2 \]
  \[ \overline{X}_{2\text{\_min}} = X_{2\text{\_min}} = 25\%, \ \overline{r}_{X_2} = 1\% \]
Fig. 6. Responses of the control system to a step decrease of $F_1$ disturbance in the 100th minute; $F_{200}$ actuator fault: not taken into consideration at all, taken into consideration, additionally the equality constraint put on $X_2$ set-point was added; above: output signals $X_2$ and $P_2$, below: control signals $P_{100}$ and $F_{200}$
**P100 actuator blockade**

- Optimizing procedure returned the message that there is no admissible solution
- Why there is no solution found?
- In order to answer this question steady–state characteristics should be analyzed
Fig. 7. Steady–state characteristics a) $X_2(F200)$ i $P_2(F200)$, b) $X_2(P100)$ i $P_2(P100)$, of the plant with blocked actuator of the manipulated variable a) $P100$, b) $F200$
Fig. 8. Responses of the control system obtained for blockade of the $P_{100}$ actuator: not taken into consideration, taken into consideration in the control system with additional manipulated variable above: output signals $X_2$ and $P_2$, below: control signals $F_3$ and $F_{200}$
Summary

• Effective and relatively little complicated method of actuator fault tolerance in control systems with MPCEO algorithms and output constraints

• The method: equality constraints added to the optimization problem solved by the algorithm

• The method can be used in the MPCEO algorithm with either linear or nonlinear dynamic control plant model

• Further improvement of control system operation can be obtained, when the additional manipulated variable is available