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Stable, effective fuzzy DMC algorithms with on–line quadratic optimization

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PLAN OF PRESENTATION:

1. Introduction
2. Fuzzy DMC algorithms based on numerical formulation
   2.1. Classical DMC algorithm
   2.2. Fuzzy DMC algorithms
3. Fuzzy DMC controller based on analytical formulation
   3.1. Classical DMC controller
   3.2. Fuzzy DMC controller
4. Stable fuzzy DMC algorithms based on numerical formulation
5. Example
6. Summary

Introduction

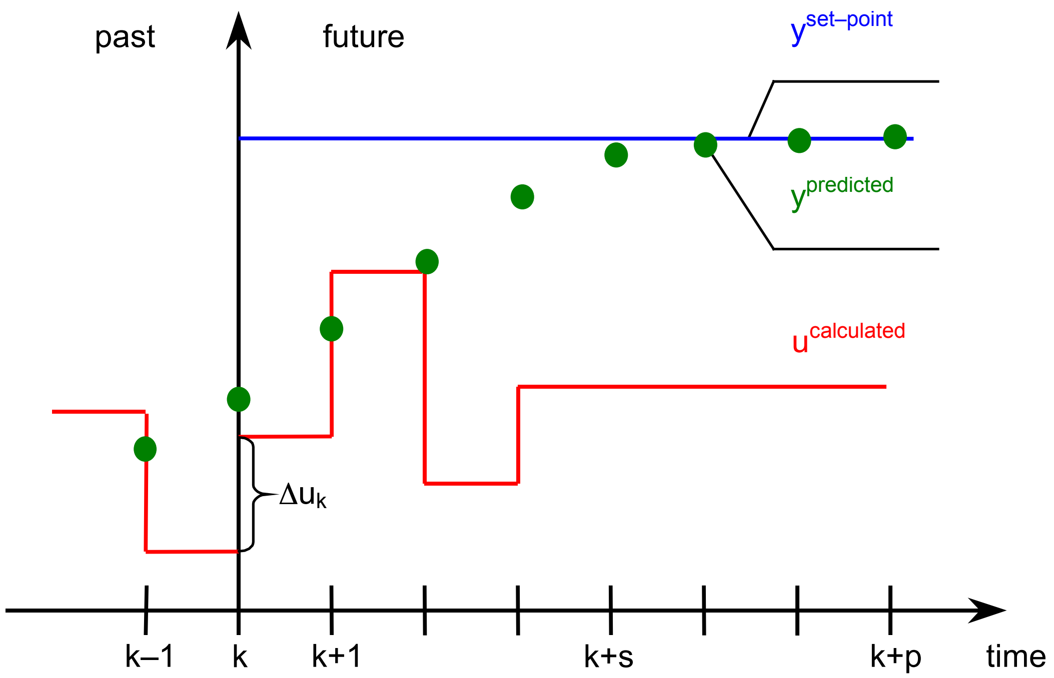

Fig. 1. Idea of predictive control p – prediction horizon, s – control horizon, Δu_k – control signal change at current iteration

Predictive control advantages:

- Possibility of taking into account:
  - — constraints,
  - — future set–point changes,
  - — anticipated disturbance changes;
- Multidimensional controllers can be designed relatively easy

Introduction

Predictive control with general nonlinear optimization:

- Huge computational burden
- Time needed cannot be anticipated
- The optimization routine may terminate in a local minimum

FDMC controllers:

- Combination of two ideas:
  — long–range horizon DMC predictive controller,
  — fuzzy Takagi–Sugeno approach
- The advantages of both techniques are included
- Only a convex quadratic programming problem is solved
- In the nonlinear case:
  — lead to better performance than algorithms based on linear models,
  — the performance is comparable to that obtained when algorithms with nonlinear optimization are used
- Stabilizing mechanism is available

Conventional DMC algorithm based on analytical formulation

The basic idea of the DMC algorithm:

$$\min_{\Delta u} \sum_{i=1}^{p} \left(y_k^{sp} - y_{k+i|k}\right)^2 + \lambda \cdot \sum_{i=0}^{s-1} \left(\Delta u_{k+i|k}\right)^2$$

where $\boldsymbol{y} = \left[y_{k+1|k}, \ldots, y_{k+p|k}\right]^T = \boldsymbol{y}^{fr} + \boldsymbol{A} \cdot \boldsymbol{\Delta u}$, $\boldsymbol{y}^{fr} = \left[y_{k+1|k}^{fr}, \ldots, y_{k+p|k}^{fr}\right]^T$,

$$\boldsymbol{\Delta u} = \left[\Delta u_{k|k}, \ldots, \Delta u_{k+s-1|k}\right]^T, \quad \boldsymbol{y}^{sp} = \left[y_k^{sp}, \ldots, y_k^{sp}\right]^T$$

A unique solution:

$$\boldsymbol{\Delta u} = \left(\boldsymbol{A}^T \cdot \boldsymbol{A} + \lambda \cdot \boldsymbol{I}\right)^{-1} \cdot \boldsymbol{A}^T \cdot \left(\boldsymbol{y}^{sp} - \boldsymbol{y}^{fr}\right)$$

Only the first element of $\boldsymbol{\Delta u}$ is used at each time step.

The structure of the controller

$$u_k = u_{k-1} + r_0 \cdot e_k + \sum_{j=1}^{p-1} r_j \cdot \Delta u_{k-j}$$

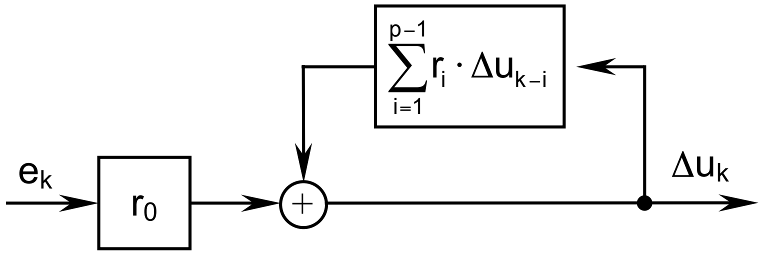

Fig. 2. Block diagram of the DMC controller

Fuzzy DMC algorithm based on analytical formulation

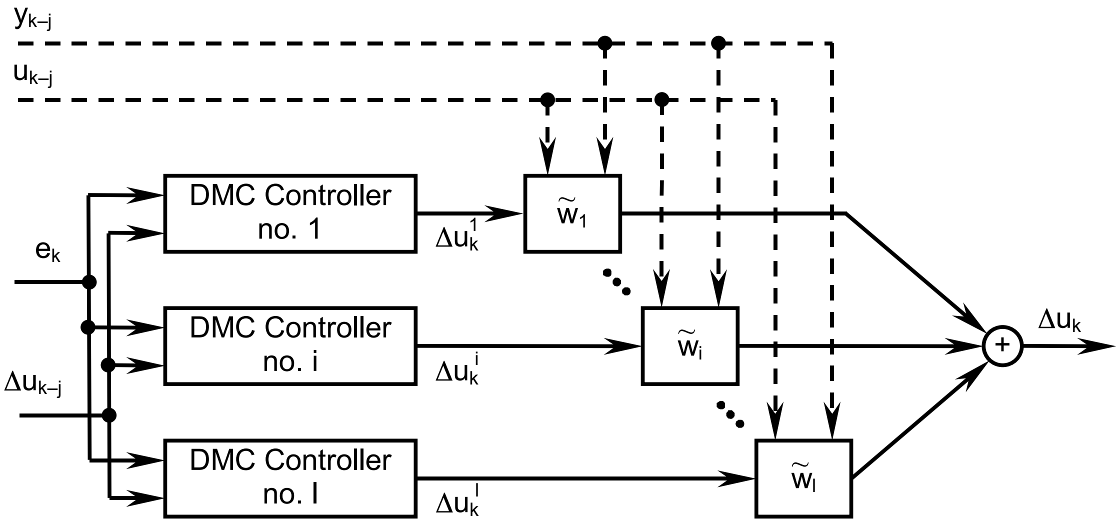

Fig. 3. Block diagram of the analytical version of the FDMC controller

- The controller is a combination of many sub–controllers.
- Parameters of sub–controllers are derived beforehand.
- Output value of the whole controller is a sum of weighted outputs of local controllers.
- Stability can be checked using appropriate transformation and Tanaka–Sugeno criterion.
- Stability is investigated by solving LMI system:

$$\boldsymbol{P} > 0,\ \boldsymbol{A}_{ij}{}^{T}\boldsymbol{P}\,\boldsymbol{A}_{ij} - \boldsymbol{P} < 0 \ \text{ dla } i,j = 1,\ldots,l$$

$$
\boldsymbol{A_{ij}} = \begin{bmatrix}
bg_1^{i,j} & bg_2^{i,j} & \dots & bg_d^{i,j} & bg_{d+1}^{i,j} & \dots & bg_{n-1}^{i,j} & bg_n^{i,j} & cf_2^{i,j} & cf_3^{i,j} & \dots & cf_{m-1}^{i,j} & cf_m^{i,j} \\
1 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 1 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \dots & 0 & 0 & \dots & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & \dots & 0 & -g_{d+1}^{j} & \dots & -g_{n-1}^{j} & -g_n^{j} & f_2^{j} & f_3^{j} & \dots & f_{m-1}^{j} & f_m^{j} \\
0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 & 0 \\
0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & 1 & \dots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 1 & 0
\end{bmatrix},
$$

$$
\text{where } bg_{in}^{i,j} = \begin{cases} b_{in}^{i}; & in < d+1 \\ b_{in}^{i} - c_1^{i} \cdot g_{in}^{j}; & in \geq d+1 \end{cases}, \quad cf_{im}^{i,j} = c_{im}^{i} + c_1^{i} \cdot f_{im}^{j}
$$

$$
(in = 1, \dots, n;\ im = 2, \dots, m).
$$

Conventional DMC algorithm based on numerical formulation

Following problem is solved at each iteration:

$$\min_{\boldsymbol{\Delta u}} \sum_{i=1}^{p} \left(y_k^{sp} - y_{k+i|k} \right)^2 + \lambda \cdot \sum_{i=0}^{s-1} \left(\Delta u_{k+i|k} \right)^2 ,$$

subject to the constraints:

$$\Delta \boldsymbol{u}_{min} \leq \Delta \boldsymbol{u} \leq \Delta \boldsymbol{u}_{max},$$

$$\boldsymbol{u}_{min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{max},$$

$$\boldsymbol{y}_{min} \leq \boldsymbol{y} \leq \boldsymbol{y}_{max}$$

Fuzzy DMC algorithms based on numerical formulation

- A few algorithms are available
- Algorithm most suitable for a given nonlinear plant can be selected
- Compromise between control performance and computation demand can be achieved

FDMC–SL (Single Linearization)

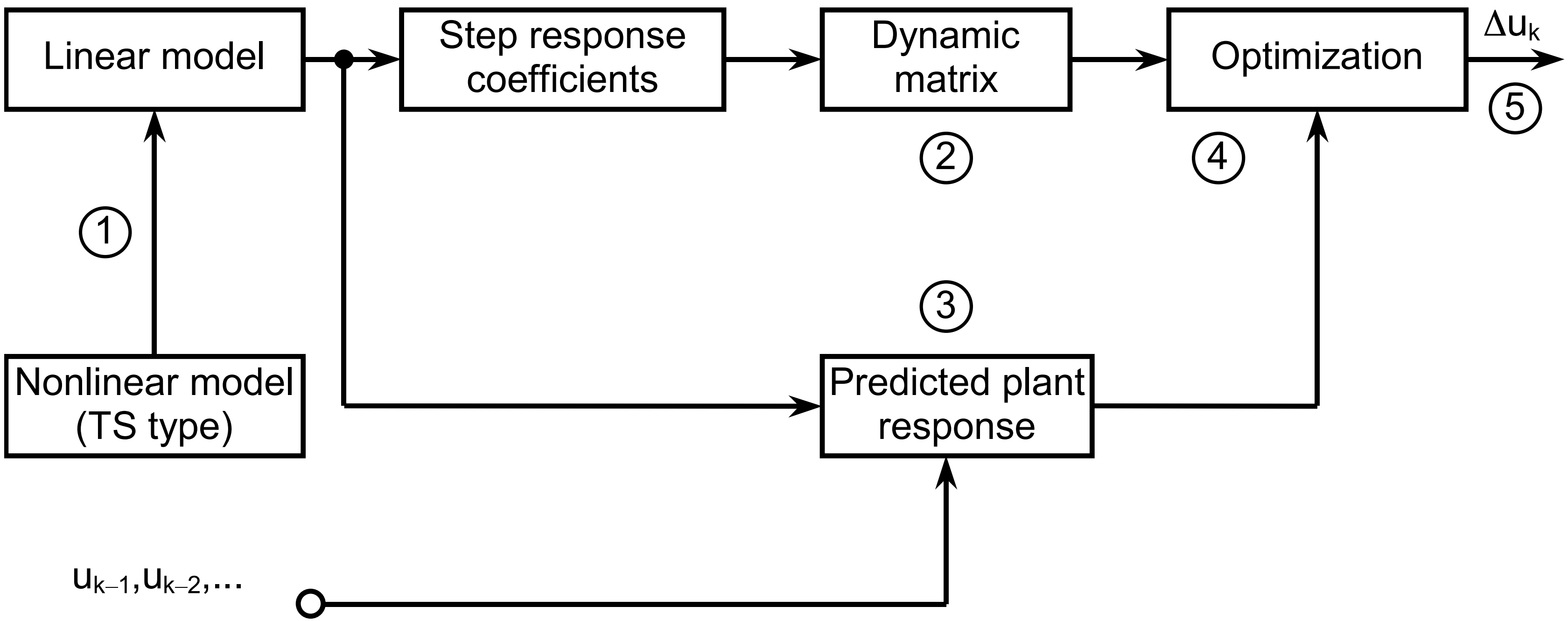

Fig. 4. Block diagram of the FDMC–SL algorithm

FDMC–SLRN (Single Linearization and Response obtained using Nonlinear model)

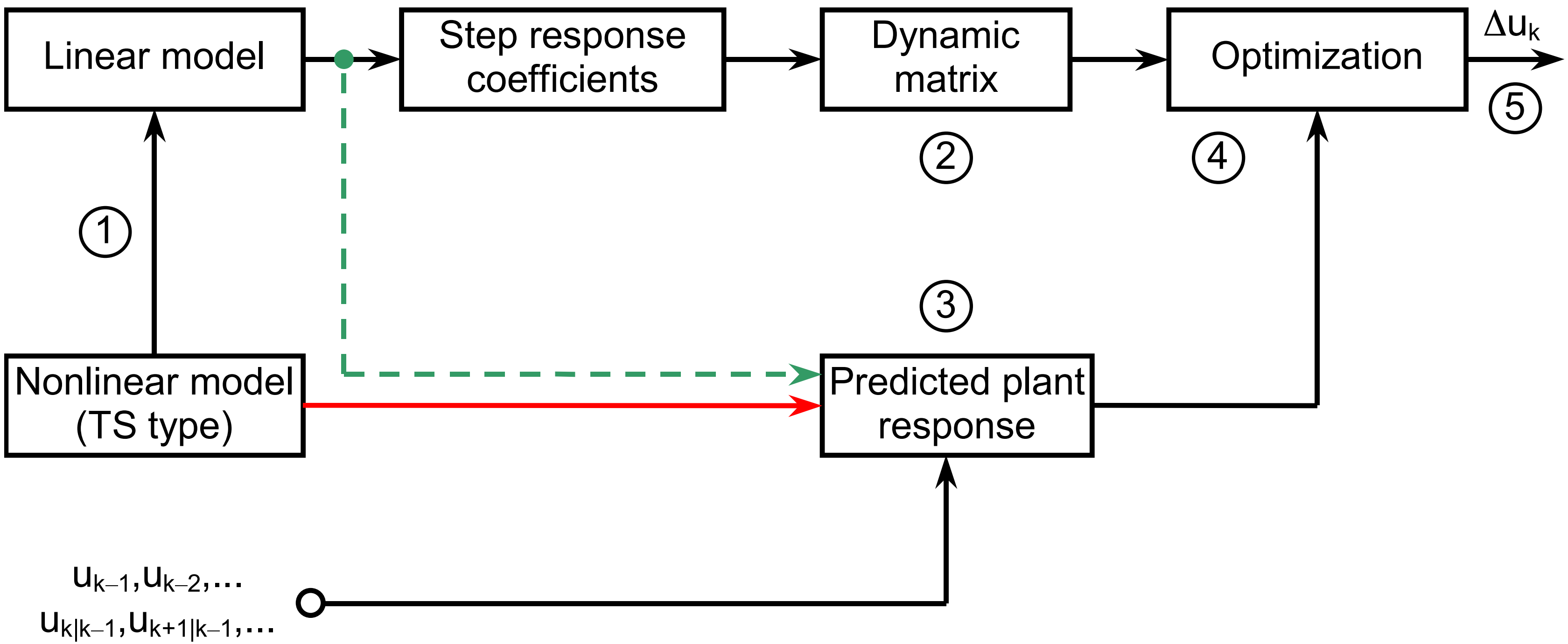

Fig. 5. Block diagram of the FDMC–SLRN (and FDMC–SL) algorithm

Stable fuzzy DMC algorithms based on numerical formulation

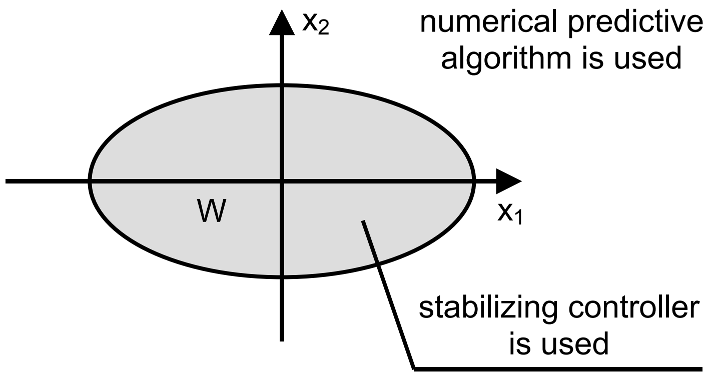

Fig. 6. The idea of dual–mode approach

The idea of dual–mode approach:

- In the set W that contains equilibrium point a stabilizing controller is used (it could be analytical FDMC controller)
- Outside the set W numerical predictive algorithm appropriately modified is used

Properties of the set W:

- Any trajectory of the control system starting in this set remains there
- The control system is asymptotically stable in this set
- The set W is inside the admissible set

Nonlinear optimization problem in dual–mode approach

$$\min_{u} \phi = \sum_{j=0}^{p-1} \theta(\bar{\boldsymbol{x}}_{k+j|k})\left(\bar{\boldsymbol{x}}_{k+j|k}^{T}\boldsymbol{Q}\bar{\boldsymbol{x}}_{k+j|k} + R(u_{k+j|k} - u_s)^2\right)$$

subject to the constraints:

$$y_{k+1} = f(y_k, u_k),$$

$$\bar{\boldsymbol{x}}_{k+p} \in W,$$

$$\Delta u_{\min} \le \Delta u_{k+j|k} \le \Delta u_{\max},$$

$$u_{\min} \le u_{k+j|k} \le u_{\max},$$

$$y_{\min} \le y_{k+j|k} \le y_{\max},$$

where $\theta(\bar{\boldsymbol{x}}_{k+j|k}) = \begin{cases} 0; \bar{\boldsymbol{x}}_{k+j|k} \in W \\ 1; \bar{\boldsymbol{x}}_{k+j|k} \notin W \end{cases}$

Problem no. 1

Stabilizing constraint $\bar{\boldsymbol{x}}_{k+p} \in W$ is nonlinear.

Problem no. 1 solution

Following constraint can be introduced to FDMC algorithms:

$$u_s = u_{k-1} + \sum_{i=0}^{s-1} (\Delta u_{k+i|k})^2$$

- Constraint relatively simple to impose
- In the case of stable control plant the state will approach the set W

Problem no. 2

In the standard approach nonlinear optimization is used to find the set *W*.

Problem no. 2 solution

Only a set of problems of the following type is solved

$$\min_{\bar{\boldsymbol{x}}_k}\left(\bar{\boldsymbol{x}}_k^T \boldsymbol{P}\bar{\boldsymbol{x}}_k\right)$$

subject to the linear equality constraint.

Then: if the solution fulfills $\hat{\boldsymbol{x}}_k^T \boldsymbol{P}\hat{\boldsymbol{x}}_k < \alpha$ then α is decreased

otherwise, next problem from the set is solved

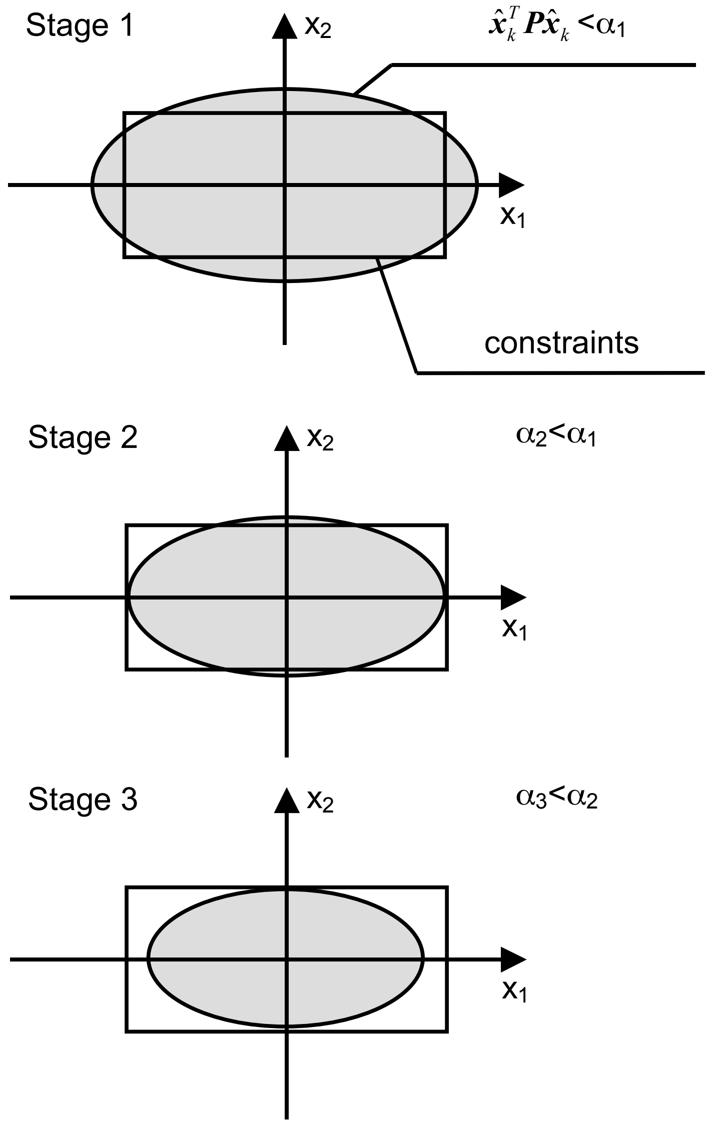

Fig. 7. Illustration of problem no. 2 solution

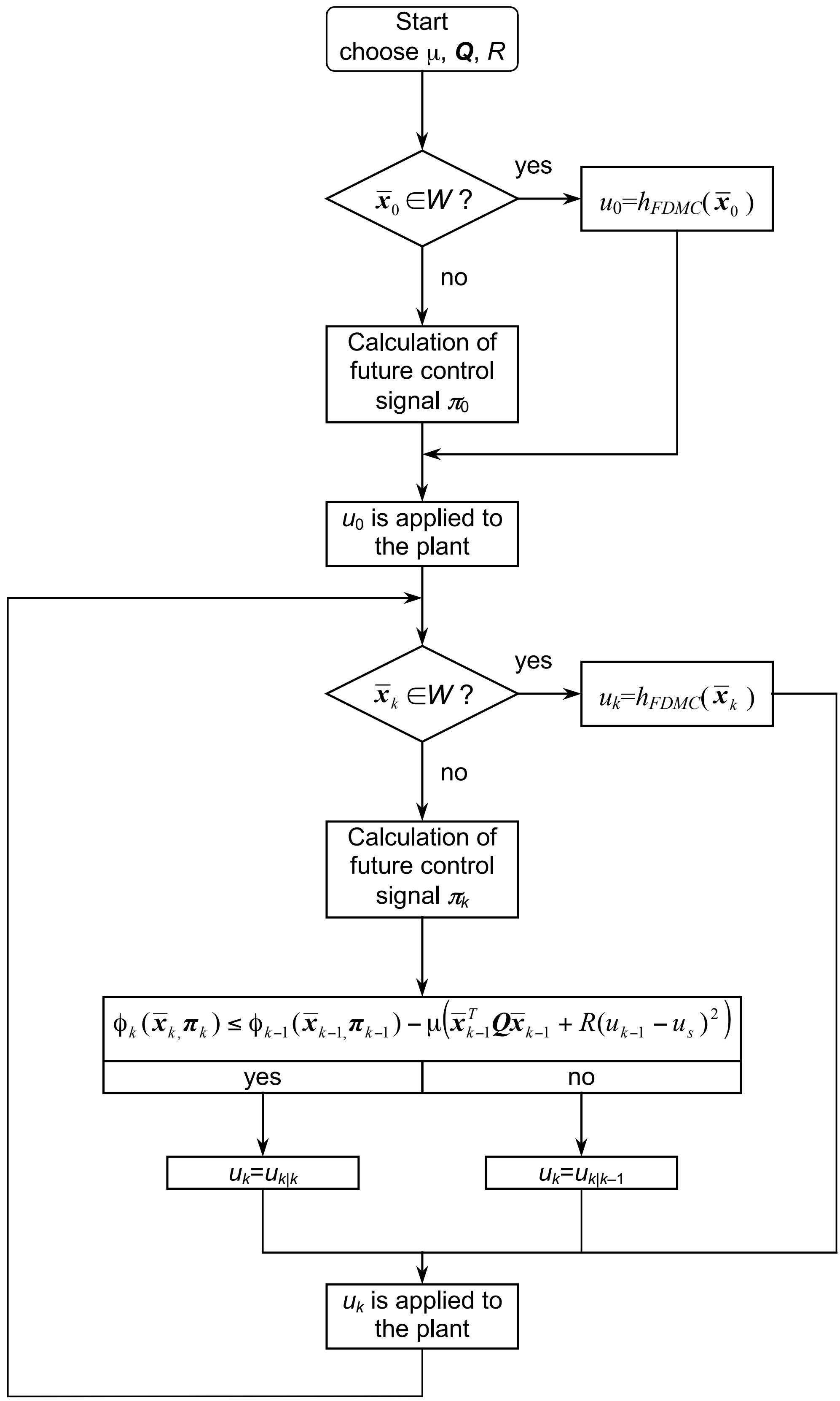

Fig. 8. Block diagram of the algorithm

Control plant

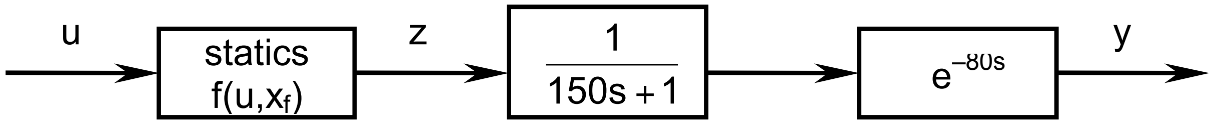

Fig. 9. Block diagram of the control plant model; u– manipulated variable, x_f – measurable disturbance, y – output variable

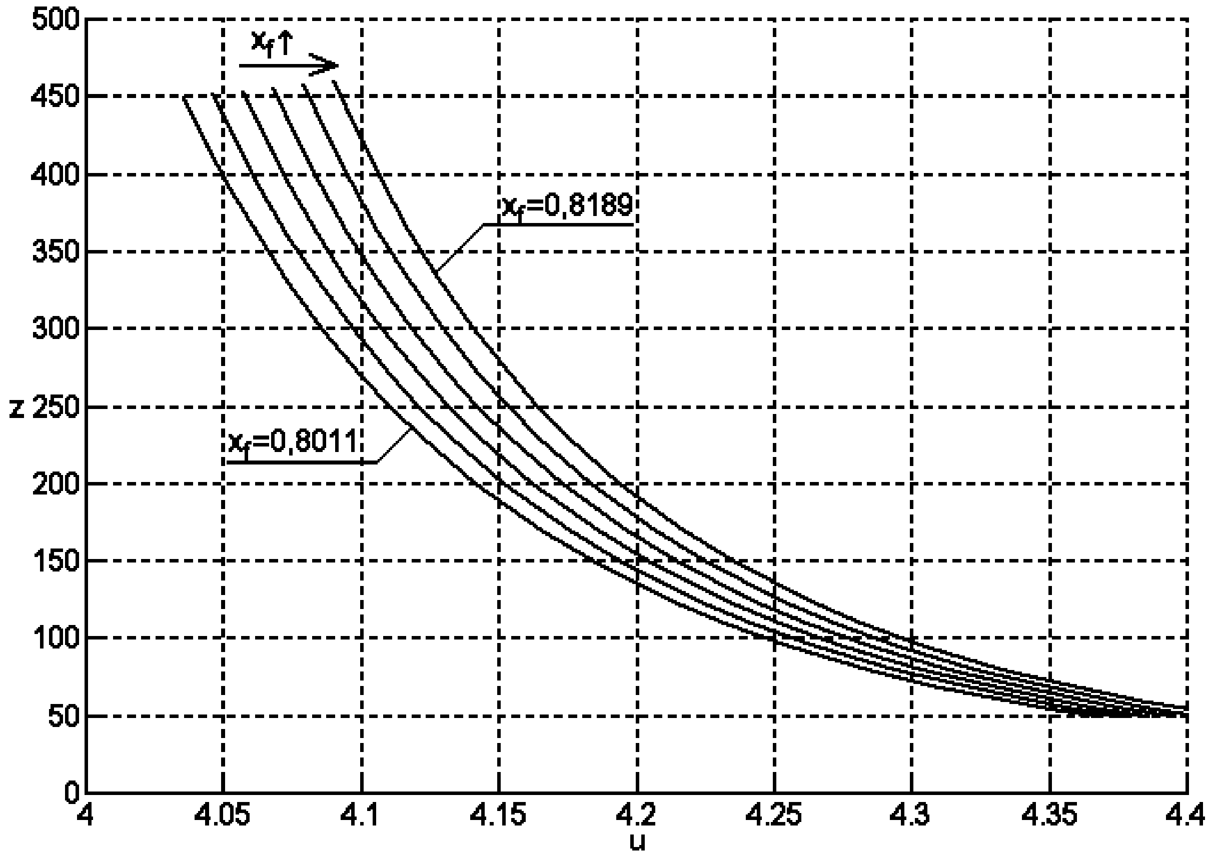

Fig. 10. Static characteristics of the control plant

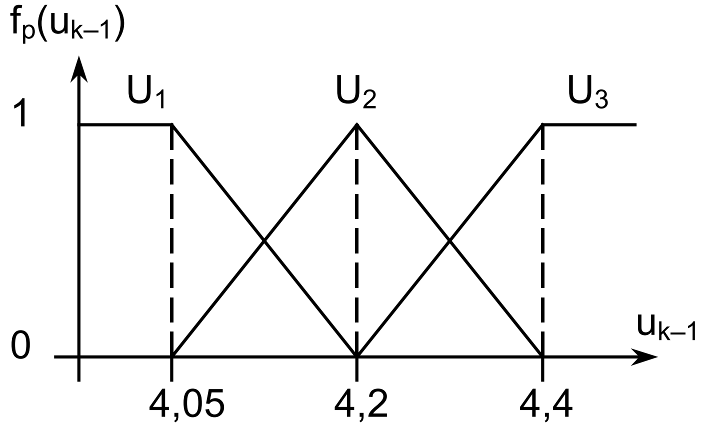

Fig. 11. Membership functions in controllers

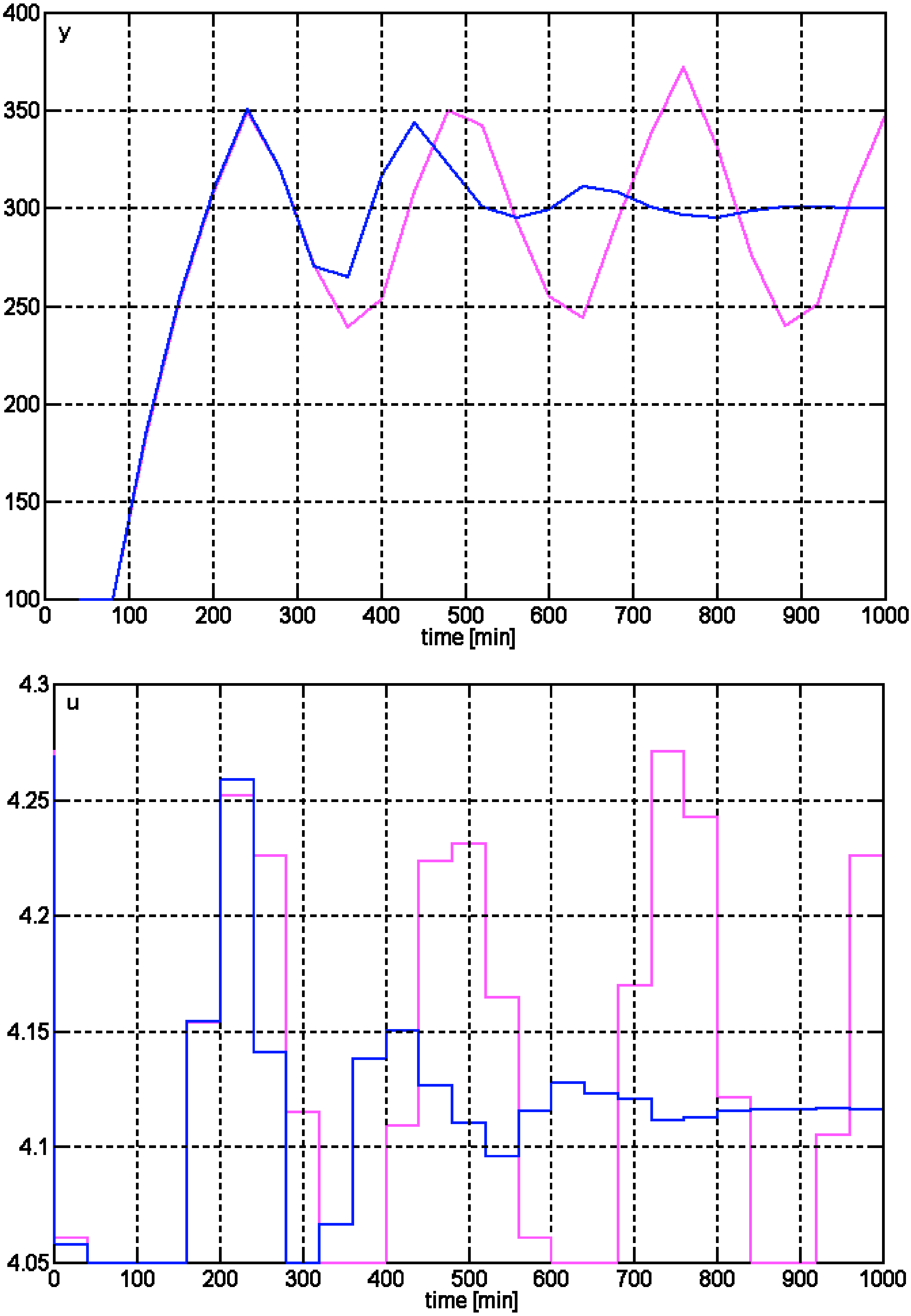

Fig. 12. Responses of the control systems with FDMC–SLRN algorithms with and without stabilizing modification to set–point change from z_0 = 100 *ppm* to z_{sp} = 300 *ppm*; above output signal, below control signal

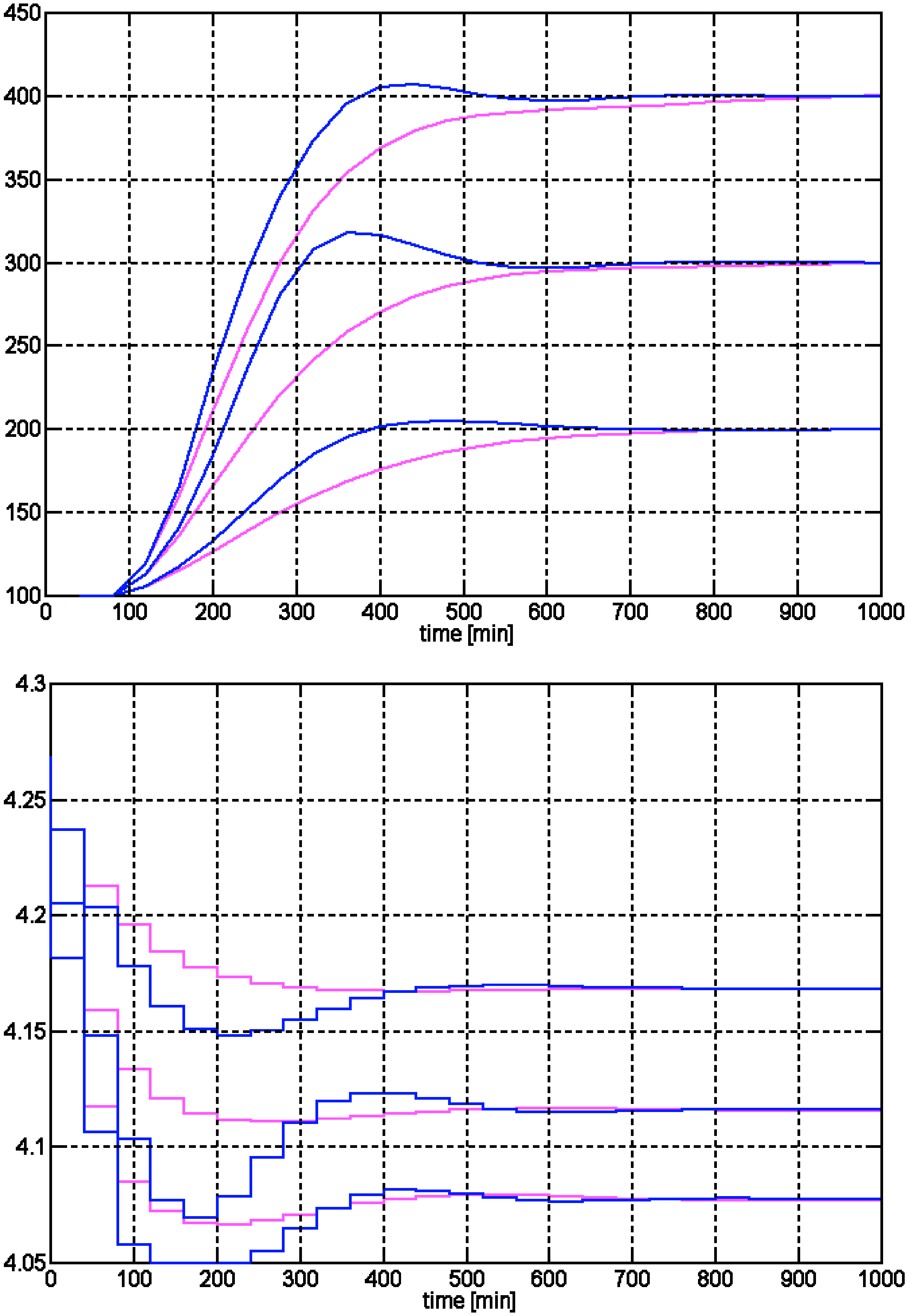

Fig. 13. Responses of the control systems with FDMC–SL and FDMC–SLRN algorithms to set–point changes from $z_0 = 100$ *ppm*; above output signal, below control signal

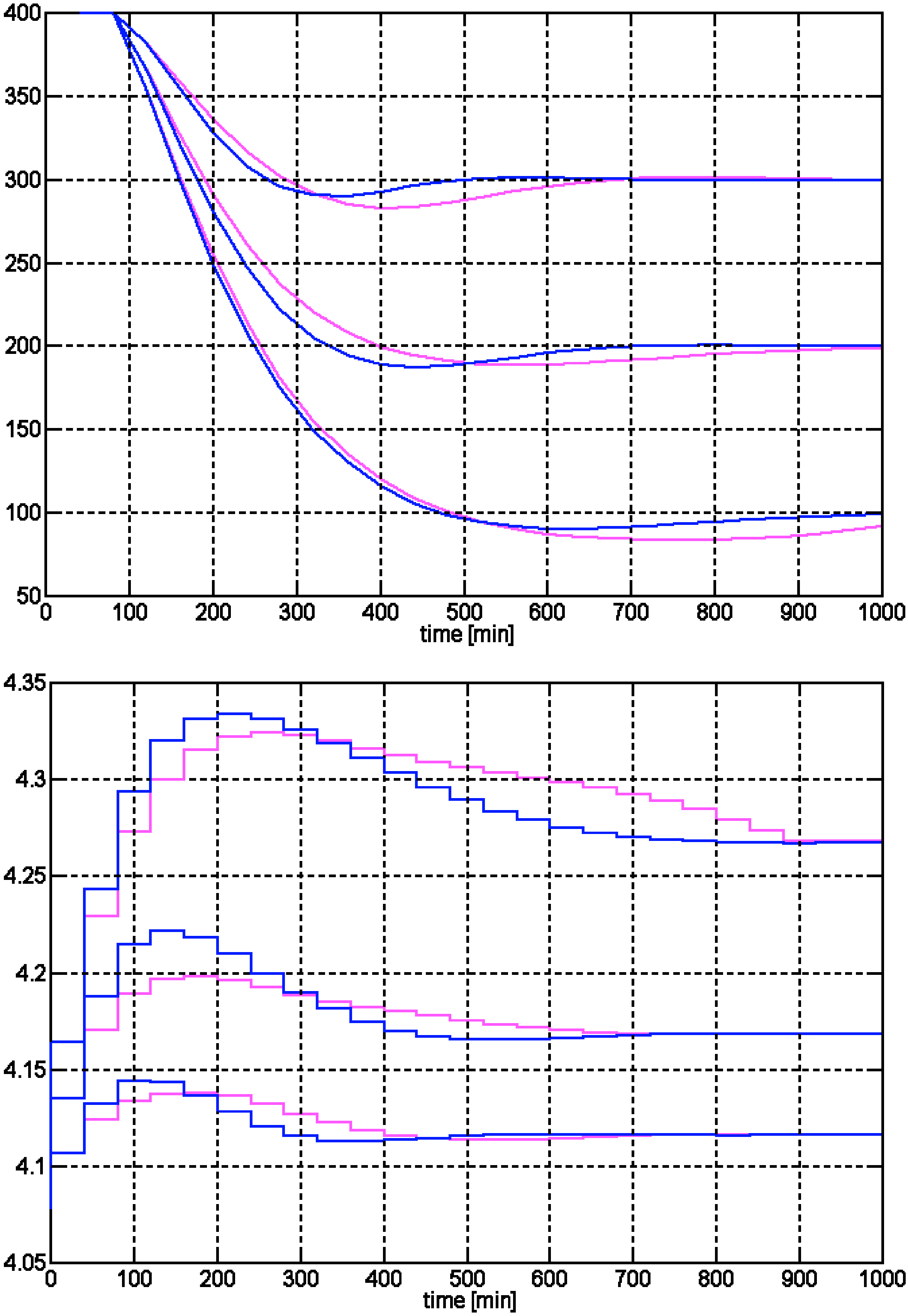

Fig. 14. Responses of the control systems with FDMC–SL and FDMC–SLRN algorithms to set–point changes from $z_0 = 400$ *ppm*; above output signal, below control signal

Summary

- Effective and relatively little complicated FDMC algorithms with stabilizing mechanism were presented
- Introduced stabilization mechanism is simple and easy to implement
- The stabilization mechanism can be used in any FDMC algorithm variant
- The most suitable algorithm can be selected depending on the nonlinearity of the plant